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Force Prediction Model for Milling 618 Stainless Steel Using Response Surface Methodology

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Abstract: This paper describes the development of a response model (cutting force) for milling 618 stainless steel utilizing response surface methodology. The cutting force model is developed in terms of cutting speed, feed rate, axial depth and radial depth. The cutting force contours have been generated from these model equations and are shown of different plots. The model generated show that the cutting force reaches the maximum value when cutting speed decreased and , feed rate, axial depth and radial depth are increased. The second order is more accurate based on the variance analysis and the predicted value is closer to the experimental result.

Key words: Cutting force, surface response methodology, first order

INTRODUCTION

In order to get the adequate model that related the cutting force and the cutting parameters (cutting speed, feed rate, axial depth and radial depth), a large number of experiments needed, that is different tests for each and every combination of cutting tool and work-piece material. In this paper, several of cutting speed, feed rate, axial depth and radial depth been takes into account and predicts the cutting force.

In this work, experimental results were used for modeling using response surface methodology (RSM)[1]. The RSM is practical, economical and relatively easy to use and it was used by a lot of researchers for modelling machining processes[2-4]. Mead and Pike[5] and Hill and Hunter[6] reviewed the earliest work on response surface methodology. Response surface methodology (RSM) is a combination of experimental and regression analysis and statistical inferences. The concept of a response surface involves a dependent variable y called the response variable and several independent variables x1, x2,. . .,xk[7].

Response model: If all of these variables are assumed to be measured, the response surface can be expressed as:

$$y = f(x1; x2; \ldots; xk) \quad (1)$$

The goal is to optimize the response variable y. It is assumed that the independent variables are continuous and controllable by the experimenter with negligible error. The response or the dependent variable is assumed to be a random variable. Say in a milling operation, it is necessary to find a suitable combination of cutting speed (x_1=ln V), feed (x_2 = ln f), axial depth (x_3 = ln a_x) and radial depth(x_4 = ln a_r) that optimize cutting force (y = ln F). The observed response y as a function of the speed, feed, axial depth and radial depth can be written as

$$y = f(x_1, x_2, x_3, x_4) + \varepsilon \quad (2)$$

Usually a low order polynomial (first-order and second-order) in some regions of the independent variables is employed. The first-order model,

$$y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \varepsilon \quad (3)$$

and the second –order model,

$$y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x^2{}_i + \sum_i \sum_j \beta_{ij} x_i x_j + \varepsilon$$
for i<j (4)

are generally utilized in RSM problems. The β parameters of the polynomials are estimated by the method of least squares.

The proposed relationship between the machining responses (cutting force) and machining independent variables can be represented by the following:

$$F = C\,(V^m f^n A_x^y A_r^z)\varepsilon' \quad (5)$$

Where F is the cutting force in N, V, f , A_x and A_r are the cutting speed (m s^{-1}), feed rate (mm rev^{-1}), axial depth (mm) and radial depth (mm). C, m, n, y and z are the constants. Equation (1) can be written in the following logarithmic form:

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$$\ln F = \ln C + m \ln V + n \ln f + y \ln A_x + z \ln A_r + \ln \varepsilon' \quad (6)$$

Equation (2) can be written as a linear form:

$$y = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \varepsilon \quad (7)$$

where, y is the cutting force, x_0 = 1(dummy variables), x_1= ln V, x_2 = ln*f* , x_3 = ln A_x , x_4 = ln A_r and ε = ln ε,where ε is assumed to be normally-distributed uncorrelated random error with zero mean and constant variance, β_0 =ln C and β_1, β_2, β_3 and β_4 are the model parameters. The second model can be expressed as:

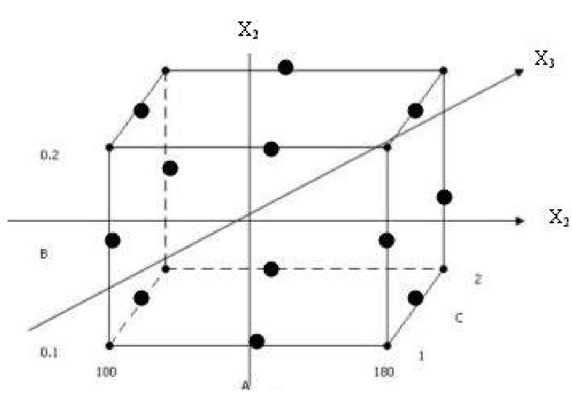
$$y'' = \beta_0 x_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{11} x^2{}_1 + \beta_{22} x^2{}_2 + \beta_{33} x^2{}_3 + \beta_{44} x^2{}_4 + \beta_{11} x_1 x_2 + \beta_{12} x_1 x_3 + \beta_{13} x_1 x_4 + \beta_{14} x_2 x_3 + \beta_{15} x_3 x_4 \quad (8)$$

The values of β_1, β_2, β_3 and β_4 are to be estimated by the method of least squares. The basic formula is:

$$(x^T x)\, \beta = x^T y \quad \beta = (x^T x)^{-1} x^T y \quad (9)$$

where, x^T is the transpose of the matrix x and $(x^T x)^{-1}$ is the inverse of the matrix $(x^T x)$. The details of the solution by this matrix approach are explained in[1,8]. The parameters have been estimated by the method of least-square using a Matlab computer package.

Experimental design: To develop the first-order, a design consisting 27 experiments was conducted. Box-Behnken Design is normally used when performing non-sequential experiments. That is, performing the experiment only once. These designs allow efficient estimation of the first and second –order coefficients. Because Box-Behnken Design has fewer design points, they are less expensive to run than central composite designs with the same number of factors. Box-Behnken Design does not have axial points, thus can be sure that all design points fall within the safe operating. Box-Behnken Design also ensures that all factors are never set at their highest levels simultaneously[9-11]. Figure 1 shows the 3 factors Box-Behnken. Preliminary tests were carried out to find the suitable cutting speed, federate, axial depth and radial depth as shown in Table 1.



Experimental details: The 618 stainless steel work-pieces were provided in the fully annealed condition in sizes of 65x170 mm and produce by Sanyo Special Steel Co. Ltd.. The tools used in this study are carbide inserts PVD coated with one layer of TiN. The inserts are manufactured by Kennametal with ISO designation of KC 735M.They are specially developed for milling applications where stainless steel is the major machined material.

Everyone passes (one pass is equal to 85mm), the cutting test were stopped. The same experiment has been repeated for 3 times to get a more accurate result. Table 2 shows the experimental cutting conditions together with the measured torque.

RESULTS AND DISCUSSION

First-order model: The cutting force first order model is:

$$y = 5.3715 - 0.1308x_1 + 0.3017x_2 + 0.2583x_3 + 0.2592x_4 \quad (10)$$

Table 3 shows the 95% confidence interval for the experiments. The analysis of variance is shown in Table 4. For the linear model, the p-value for lack of fit is 0.144 and the F-statistics are 6.38. Therefore, the model is adequate.

The levels of independent variables and coding identifications used in this design are presented in Table 1. Table 2 shows the experimental conditions and results obtained from experiments. The transforming equations for each of the independent variables are:

$$x_1 = \frac{\ln(V) - \ln(v)centre}{\ln(v)high - \ln(v)centre}$$

$$x_2 = \frac{\ln(f) - \ln(f)centre}{\ln(f)high - \ln(f)centre}$$

$$x_3 = \frac{\ln(A_x) - \ln(a_x)centre}{\ln(a_x)high - \ln(a_x)centre}$$

$$x_4 = \frac{\ln(A_r) - \ln(a_r)centre}{\ln(a_r)high - \ln(a_r)centre} \quad (11)$$

Equation (10) describing the cutting force model can be transformed using Equation (11) into the following form:

$$F = 5734.547(V^{-0.52049}F^{1.0487}A_x^{0.89787}A_r^{0.726722}) \quad (12)$$

Table 1: Levels of independent variables

Levels	Low	Medium	High
Coding	-1	0	1
Speed v (m s^{-1})	100	140	180
Feed f (mm rev^{-1})	0.1	0.2	0.3
Axial depth d_a (mm)	1	1.5	2
Radial depth d_r (mm)	2	3.5	5

Table 2: Experiment condition and results

Run	Cutting speed(m s^{-1})	Feed(mm rev^{-1})	Axial depth(mm)	Radial depth(mm)	Exp.Force(N)
1	140	0.15	1	2	146.67
2	140	0.2	1	3.5	190
3	100	0.15	1	3.5	190
4	180	0.15	1	3.5	170
5	140	0.1	1	3.5	110
6	140	0.15	1	5	225
7	100	0.15	1.5	2	240
8	140	0.1	1.5	2	100
9	100	0.2	1.5	3.5	340
10	140	0.15	1.5	3.5	220
11	180	0.2	1.5	3.5	293.33
12	180	0.15	1.5	2	145
13	140	0.2	1.5	2	200
14	140	0.15	1.5	3.5	325
15	140	0.15	1.5	3.5	200
16	180	0.1	1.5	3.5	130
17	100	0.1	1.5	3.5	190
18	100	0.15	1.5	5	340
19	140	0.1	1.5	5	210
20	180	0.15	1.5	5	240
21	140	0.15	1.5	3.5	200
22	140	0.15	2	5	350
23	140	0.2	2	3.5	350
24	140	0.1	2	3.5	200
25	140	0.15	2	2	190
26	100	0.15	2	3.5	340
27	180	0.15	2	3.5	313.33

Table 3: The predicted result of the first order model

Run	Cutting speed(m s^{-1})	Feed rate(mm rev^{-1})	Axial depth(mm)	Radial depth(mm)	Exp. Force (N)	Pre. Force (N)
2	140	0.15	1	2	146.67	99.13
7	140	0.2	1	3.5	190	201.30
11	100	0.15	1	3.5	190	177.37
14	180	0.15	1	3.5	170	130.62
19	140	0.1	1	3.5	110	97.31
21	140	0.15	1	5	225	192.92
4	100	0.15	1.5	2	240	169.96
5	140	0.1	1.5	2	100	93.25
6	100	0.2	1.5	3.5	340	345.15
9	140	0.15	1.5	3.5	220	214.25
10	180	0.2	1.5	3.5	293.33	254.18
12	180	0.15	1.5	2	145	125.17
15	140	0.2	1.5	2	200	192.90
22	140	0.2	1.5	5	325	375.42
24	140	0.15	1.5	3.5	200	214.25
25	180	0.1	1.5	3.5	130	122.87
26	100	0.1	1.5	3.5	190	166.84
8	100	0.15	1.5	5	340	330.79
17	140	0.1	1.5	5	210	181.48
18	180	0.15	1.5	5	240	243.60
22	140	0.15	1.5	3.5	200	214.25
1	140	0.15	2	5	350	359.48
3	140	0.2	2	3.5	350	375.08
13	140	0.1	2	3.5	200	181.31
16	140	0.15	2	2	190	184.71
20	100	0.15	2	3.5	340	330.49
27	180	0.15	2	3.5	313.33	243.38

Table 4: Analysis of variance for Force Fy

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	4	137461	137461	34365.2	43.77	0.000
Linear	4	137461	137461	34365.2	43.77	0.000
Residual Error	22	17272	17272	785.1		
Lack-of-Fit	20	17005	17005	850. 3	6.38	0.144
Pure Error	2	267	267	133.3		
Total	26	154733				

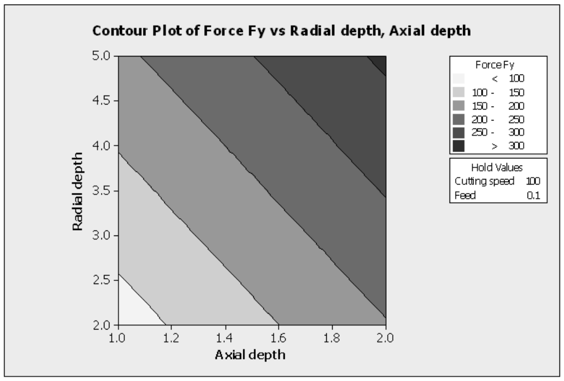


Fig. 2a: Cutting force contours in the Axial depth-radial depth plane for cutting speed 100 m s^{-1} and feed rate 0.1 mm rev^{-1}

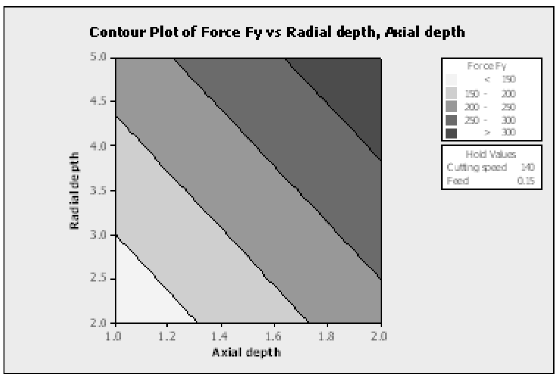


Fig. 2b: Cutting force contours in the Axial depth-radial depth plane for cutting speed 140 m s^{-1} and feed rate 0.15 mm rev^{-1}

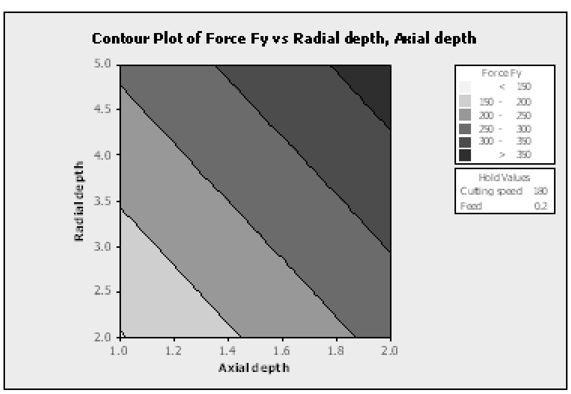


Fig. 2c: Cutting force contours in the Axial depth-radial depth plane for cutting speed 180 m s^{-1} and feed rate 0.2 mm rev^{-1}

This result shows that feed rate has the most significant effect on the cutting force, follow by axial depth, radial depth and cutting speed. The equation shows that the cutting force increasing with reducing the cutting speeds and increasing the feed rate, axial depth and radial depth.

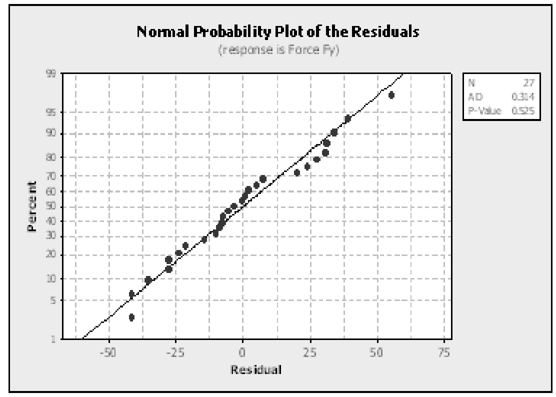


Fig. 3: Normal probability plot of the residuals

Equation (10) is utilized to develop cutting force contour at the selected cutting speed and feed rate. Figure 2 (a) to 2 (c) show the cutting force contour with selected axial depth and radial depth. These contours help to predict the cutting force at any zone of experimental zone. Figure 3 shows that the residual plot is fit to the normal line.

From the contour, the cutting force reaches the highest value at Fig. 1 (c) where the value of cutting speed at its lower value, feed rate, axial depth and radial depth at their maximum value. The cutting force can reach more than 350N in Fig. 1 (c) .The lowest cutting force is in Fig. 1 (a) when the cutting speed at its maximum value and the other factors at its maximum value. From this contour plot, the safety zone of cutting force can be selected for any experiment.

The second-order model was postulated in obtaining the relationship between the cutting force and the machine independent variables. The model was based on the Box-Behnken Design method. The model equation is:

$$y'' = 2.05074 - 0.031x_1 + 47.37x_2 + 2.97x_3 + 1.60x_4 + 0.00029x^2{}_1$$
$$-50.17x^2{}_2 - 0.78x^2{}_3 - 0.14x^2{}_4 - 0.29x_1x_2 - 0.018x_1x_3 - 0.0094x_1x_4$$
$$+24.3x_2x_3 + 12.8x_2x_4 + 0.80x_3x_4$$

Table 5 shows the 95% confidence interval for the experiments. The analysis of variance is shown in Table 6. For the second-order model, the p-value for lack of fit is 0.221 and the F-statistics are 4.5249. Therefore, the model is adequate. The second-order model is more adequate, because the predicted result is much more accurate than the first model. The p-value show much bigger than the first order. Equation (8) is used to develop the contour plot as shown in Fig. 4 (a) to 4 (c). Figure 5 shows that the residual plot is fit to the normal line.

Table 5: The predicted result of the second order model

Run	Cutting speed	Feed rate	Axial depth	Radial depth	Exp.Force(N)	Pre.Force(N)
2	140	0.15	1	2	146.67	130.56
7	140	0.2	1	3.5	190	202.36
11	100	0.15	1	3.5	190	220.69
14	180	0.15	1	3.5	170	165.97
19	140	0.1	1	3.5	110	110.97
21	140	0.15	1	5	225	201.11
4	100	0.15	1.5	2	240	216.81
5	140	0.1	1.5	2	100	92.92
6	100	0.2	1.5	3.5	340	324.45
9	140	0.15	1.5	3.5	220	206.67
10	180	0.2	1.5	3.5	293.33	273.05
12	180	0.15	1.5	2	145	161.25
15	140	0.2	1.5	2	200	211.81
22	140	0.2	1.5	5	325	330.69
24	140	0.15	1.5	3.5	200	206.67
25	180	0.1	1.5	3.5	130	140.00
26	100	0.1	1.5	3.5	190	204.72
8	100	0.15	1.5	5	340	330.70
17	140	0.1	1.5	5	210	196.81
18	180	0.15	1.5	5	240	270.14
22	140	0.15	1.5	3.5	200	206.67
1	140	0.15	2	5	350	360.56
3	140	0.2	2	3.5	350	355.97
13	140	0.1	2	3.5	200	194.58
16	140	0.15	2	2	190	208.33
20	100	0.15	2	3.5	340	342.64
27	180	0.15	2	3.5	313.33	281.25

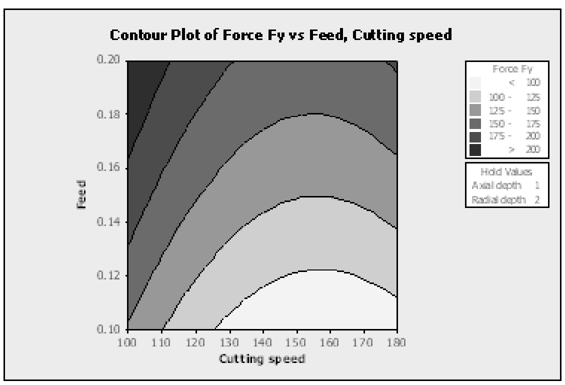


Fig. 4a: Cutting force contours in the Feed rate-cutting speed plane for Axial depth 1 mm and radial depth 2 mm

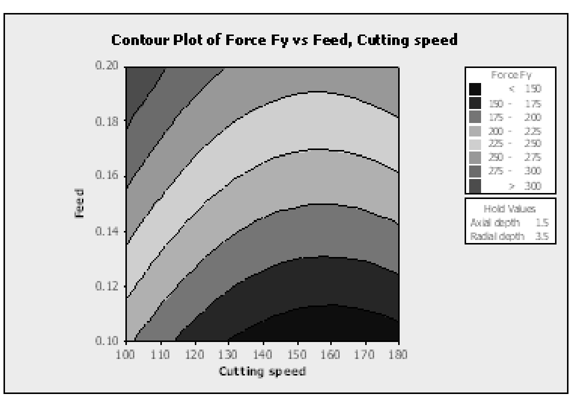


Fig. 4b: Cutting force contours in the feed rate-cutting speed plane for Axial depth 1.5 mm and radial depth 3.5 mm

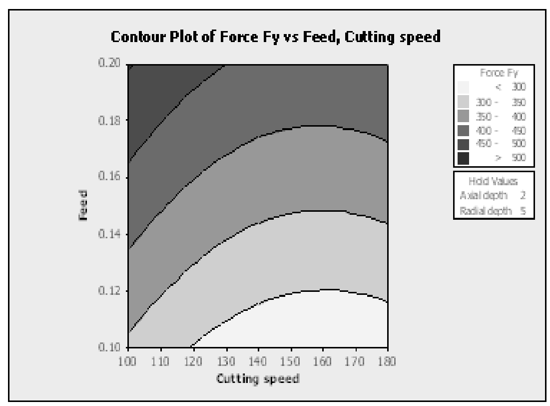


Fig. 4c: Cutting force contours in the feed rate-cutting speed plane for Axial depth 2mm and radial depth 5mm

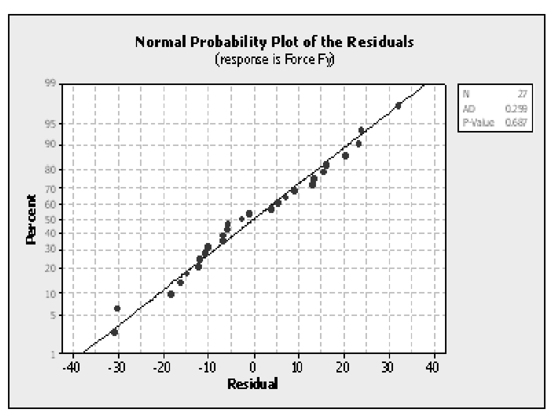


Table 6: Analysis of variance for second-order model

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Regression	14	447.358	447.358	31.954	1758.88	0.000
Linear	4	434.746	434.746	108.687	5982.52	0.000
Square	4	2.922	2.922	0.731	40.21	0.000
Interaction	6	9.690	9.690	1.615	88.90	0.000
Residual Error	12	0.218	0.218	0.018		
Lack-of-Fit	10	0. 218	0.218	0.022	4.5249	0.221
Pure Error	2	0.000	0.000	0.00486		
Total	26	447.576				

CONCLUSION

Reliable cutting force model has been developed and utilized to enhance the efficiency of the milling 618 stainless steel. The cutting force equation shows that feed rate, cutting speed, axial depth and radial depth play the major role to produce the cutting force. The higher the feed rate, axial depth and radial depth, the cutting force generates very high compared with low value of feed rate, axial depth and radial depth. The contours of the cutting force outputs were constructed in planes containing two of the independent variables. These contours were further developed to select the proper combination of cutting speed, feed, axial depth and radial depth to produce the optimum cutting force. Response surface methodology provides a large amount of information with a small amount of experimentation.

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