American Journal of Applied Sciences 8 (2): 113-115, 2011
ISSN 1546-9239

New Sufficient Condition of Discrete-Time Systems of Neural Networks

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Abstract: Problem statement: In this study, we derive a new sufficient condition for asymptotic stability of the zero solution of delay-differential system of neural networks in terms of certain matrix inequalities by using a discrete analog of the new Lyapunov second method. **Conclusion/Recommendations:** The problem is solved by applying a novel Lyapunov functional and an improved delay-dependent stability criterion is obtained in terms of a linear matrix inequality.

Key words: Asymptotic stability, delay-differential system, neural networks, lyapunov function, matrix inequalities, discrete-time systems

INTRODUCTION

In recent decades, neural networks have been extensively studied in many aspects and successfully applied to many fields such as pattern identifying, voice recognizing, system controlling, signal processing systems, static image treatment and solving nonlinear algebraic system (Alfaris *et al*., 2008; Bay and Phat, 2002; Bezzarga and Bucur, 2005; El-Said and EL-Sherbeny, 2005; Lekhmissi, 2006; Sen, 2004; 2005a; 2005b; Sen *et al*., 2005; Sharif and Saad, 2005; Waziri *et al*., 2005). Such applications are based on the existence of equilibrium points and qualitative properties of systems. In electronic implementation, time delays occur due to some reasons such as circuit integration, switching delays of the amplifiers and communication delays, etc. Therefore, the study of the asymptotic stability of neural networks with delays is of particular importance to manufacturing high quality microelectronic neural networks.

In this study, we consider delay-differential system of neural networks of the form

$$u(k+1) = -Cu(k) + BS(u(k-h)) + f \qquad (1)$$

where $u(k) \in \Omega \subseteq \mathbb{R}^n$ is the neuron state vector, $h \geq 0$, $C = \text{diag}\{c_1, \ldots, c_n\}$, $c_i \geq 0$, $i = 1, 2, ..., n$ is the $n \times n$ constant relaxation matrix, $A \multimap$ is the $n \times n$ constant weight matrices, $f = (f_1, \ldots, f_n) \in \mathbb{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1[\mathbb{R}, (-1,1)]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, ..., n$.

The asymptotic stability of the zero solution of the delay-differential system of neural networks has been developed during the past several years. Much less is known regarding the asymptotic stability of the zero solution of the delay-differential system of neural networks. Therefore, the purpose of this study is to establish new sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

Preliminaries: Lemma 1 (Agarwal, 1992) the zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x(k)) : \mathbb{R}^n \to \mathbb{R}^+$ such that:

$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \leq -\beta \|x(k)\|^2$$

along the solution of the system. In the case the above condition holds for all $x(k) \in V_\delta$, we say that the zero solution is locally asymptotically stable.

We present the following technical lemmas, which will be used in the proof of our main result.

Improved stability criterion: In this section, we consider the new sufficient condition for asymptotic stability of the zero solution u^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $u^* = 0, S(0) = 0$ and $f = 0$ (for otherwise, we let $x = u - u^*$ and define $S(x) = S(x + u^*) - S(u^*))$.

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Throughout this study we assume the neuron activations $s_i(x_i)$, $i = 1, 2, \ldots, n$ is bounded and monotonically nondecreasing on $\mathbb{R}$ and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i > 0$, $i = 1, 2, \ldots, n$ such that:

$$|s_i(v_1) - s_i(v_2)| \le l_i |v_1 - v_2|, \ \forall v_1, v_2 \in \mathbb{R} \tag{2}$$

By condition (2), $s_i(x_i)$ satisfy

$$|s_i(x_i)| \le l_i |x_i|, \ \ i = 1, 2, ..., n \tag{3}$$

Where:

$x(k) \in \mathbb{R}^n$ = The state vector
C and B = Known constant matrices
h (k>0) = A time-varying delay satisfying 0<h (k)≤h
h = A positive integer

MATERIALS AND METHODS

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$$\exists \beta > 0 : \Delta V(x(k)) = V(x(k+1)) - V(x(k)) \le -\beta \|x(k)\|^2$$

along the solution of the system. In the case the above condition holds for all x (k)∈V_δ, we say that the zero solution is locally asymptotically stable.

RESULTS AND DISCUSSION

Theorem 3.1 The zero solution of the delay-differential system (1) is asymptotically stable if there exist symmetric positive definite matrices P, G, W and $L = \text{diag}[l_1, \ldots, l_n] > 0$ satisfying the following matrix inequalities:

$$\psi = \begin{pmatrix} C^T PC + hG - P & C^T PBL \\ LB^T PC & LB^T PBL \end{pmatrix} < 0 \tag{4}$$

Proof Consider the Lyapunov function $V(y(k)) = V_1(y(k)) + V_2(y(k))$, where:

$$V_1(y(k)) = x^T(k)Px(k),$$

$$V_2(y(k)) = \sum_{i=k-h(k)}^{k-1} (h(k) - k + i)x^T(i)Gx(i)$$

P, G being symmetric positive definite solutions of (4) and $y(k) = [x(k), x(k - h(k))]$.

Then difference of $V(y(k))$ along trajectory of solution of (1) is given by:

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k))$$

Where:

$$\begin{aligned} \Delta V_1(y(k)) &= V_1(x(k+1)) - V_1(x(k)) \\ &= [-Cx(k) + BS(x(k - h(k)))]^T P \\ &\times[-Cx(k) + BS(x(k - h(k)))] - x^T(k)Px(k) \\ &= x^T(k)[C^T PC - P]x(k) \\ &-x^T(k)C^T PBS(x(k - h(k))) \\ &-S^T(x(k - h(k)))B^T PCx(k) \\ &+S^T(x(k - h(k)))B^T PBS(x(k - h(k))), \end{aligned}$$

$$\begin{aligned} \Delta V_2(x(k)) &= \Delta\left(\sum_{i=k-h(k)}^{k-1} (h(k) - k + i)x^T(i)Gx(i)\right) \\ &= \sum_{i=k-h(k)+1}^{k} (h(k) - (k+1) + i)x^T(i)Gx(i) \\ &- \sum_{i=k-h(k)}^{k-1} (h(k) - k + i)x^T(i)Gx(i), \end{aligned} \tag{5}$$

where (3) is utilized in (5), respectively.

Then we have:

$$\begin{aligned} \Delta V \le\ & x^T(k)[C^T PC + hG + LA^T PAL - P]x(k) \\ & -x^T(k)[C^T PAL + C^T PBL + LA^T PBL]x(k - h(k)) \\ & -x^T(k - h(k))[LA^T PC + LB^T PC + LB^T PAL]x(k) \\ & +x^T(k - h)LB^T PBLx(k - h(k)) \end{aligned}$$

where $y(k) = [x(k), x(k - h)]$ and:

$$\psi = \begin{pmatrix} C^T PC + hG - P & C^T PBL \\ LB^T PC & LB^T PBL \end{pmatrix}.$$

On the above estimation we use: $h(k) \le h$, and $h(k)x^T(k)Gx(k) \le x^T(k)Gx(k)$. By the condition (4), $\Delta V(y(k))$ is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \le -\beta\|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from Lemma 1. This completes the proof.

Remark 1: Theorem 1 gives a sufficient condition for the asymptotic stability of delay-difference system (1) via matrix inequalities. These conditions are described in terms of certain symmetric matrix inequalities But Wahab and Mohamed (2008) these conditions are described in terms of certain nonsymmetric matrix inequalities.

CONCLUSION

In this study, an improved delay-dependent stability condition for discrete-time linear systems with interval-like time-varying delays has been presented in terms of an LMI.

ACKHOWLEDGMENT

This study was supported by the Thai Research Fund Grant, the Higher Education Commission and Faculty of Science, Maejo University, Thailand.

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