

A HIGH B_1 FIELD HOMOGENEITY GENERATION USING FREE ELEMENT ELLIPTICAL FOUR-COIL SYSTEM

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ABSTRACT

Design and testing of a prototype probe with high B_1 magnetic field homogeneity that can be used for MRI is presented. It consists of four coaxial separately tuned rings of wire in a symmetric arrangement on an ellipsoidal surface. Compared to Helmholtz pair, which presents 2nd-order homogeneity, an improvement in field homogeneity is achieved, while preserving the simplicity of implementation. The proposed probe has an identical self-resonance frequency for all coils to allow easy tuning. The position and diameter of each ring are optimized so that the assembly produces a 6th-order homogeneous field. A complete electrical model of the four-coil system taking into account all coupling effects permits to determine the resonance frequency in the co-current mode and consequently to predict the value of the tuning capacitors. Resulted field homogeneity of the proposed four-coils are compared to that of Helmholtz coil which shows an improvement of 58% in field homogeneity profile when considering 10% profile uniformity width. Moreover, compared to the homologous Helmholtz coil, the proposed four-coil prototype possesses a better quality factor and therefore leads to an improved Nuclear Magnetic Resonance (NMR) sensitivity.

Keywords: Magnetic Flux Density, Field Homogeneity, Helmholtz Coil

1. INTRODUCTION

Helmholtz coil have many applications in different fields which involves generation of homogenous and controllable magnetic field. Magnetic hyperthermia in cancer therapy (Di Barba *et al.*, 2010), Magnetic Resonance Imaging (MRI) (Pulyer and Hrovat, 2002), plasma physics experiments (Anderson, 1999), calibration of magnetic fields (Bongiraud *et al.*, 1999) are some examples of applications. Air-core coils have been used in the last years for this respect to generate a uniform magnetic field within a specific space (Herceg *et al.*, 2009). In NMR experiments, an RF inhomogeneity problem makes it difficult to do a reliable quantification of metabolites by NMR spectroscopy and leads to variations of intensity across an image in MRI. Consequently the improvement of the B_1 field

homogeneity is becoming important research topic. It is of importance to define the level of magnetic field homogeneity required since this is related to the geometry of the coil used (Azpurua, 2012). In general, MRI coils with high B_1 field homogeneity and sensitivity are built with multiple radiating elements. In particular, it is already known that the addition of further coaxial rings in a Helmholtz-type design can improve homogeneity (Hoult and Deslauriers, 1990). Furthermore several researches introduced what is referred to as coaxial multi-ring coils. Additionally and beyond the traditional cylindrical structures, some research introduced spherical coils, where they prove to be very efficient in terms of field homogeneity (Harpen, 1991; Ould Ahmed Ghaly *et al.*, 2005a), however the absence of periodicity comparatively to birdcage coils at the geometric and electrical levels in addition to the more or

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less great number of tuning capacitors makes coil tuning quite difficult (Hayes *et al.*, 1985).

This article presents a description of the design of high B_1 field homogenous coil generator, comprising four coaxial separately tuned rings of wire assembled symmetrically on an ellipsoidal surface. A sixth-order homogeneous field is produced using the proposed four-coil system, compared to second-order homogeneous field by Helmholtz pair, while presenting simplicity of operation. To achieve this, an identical self-resonance frequency is chosen for all coils to allow easy tuning. A complete electrical model of the four-coil system taking into account all the coupling effects will enable us to carry out the conditions of a correct operation in co-current mode which will be the mode of interest for the proposed application.

2. ELECTRICAL MODELING

It is necessary to analyze the complete electrical model of the system in order to explain the operation and consequently prove the feasibility of the probe. The basic structure of the developed probes is schematically represented in **Fig. 1**. The coupling of the rings of the coil shall include complete inductive and capacitive coupling between them, which makes it possible to study the electric behavior of the coil. The rings are characterized by their self and mutual inductances between each other, which depend, at high frequencies, on the geometrical dimensions. The inductance L_i of any ring i can be approximated as shown in Equation (1) (Feynman and Sands, 1996):

$$L_i \approx a_i \mu_0 \left\{ \ln \left(8 \frac{a_i}{b_i} \right) - 2 \right\} \quad (1)$$

where, a_i is the radius of the ring and b_i is the radius of the wire. Also, the mutual inductance M_{ij} between two closed circuits i and j is expressed in Equation (2):

$$M_{ij} = \frac{\mu_0}{4\pi} \oint \left[\oint \frac{ds_i}{d_{ij}} \right] ds_j \quad (2)$$

where, μ_0 is the permeability of free space, ds_i and ds_j are elements of circuit i and j respectively and d_{ij} is a distance between them.

In the case of two coaxial single-turn circular loops of wire, using elliptic integrals, Equation (2) was solved in (Feynman and Sands, 1996) according to the geometrical dimensions as shown in Equation (3):

$$M_{ij} = \mu_0 (a_i a_j)^{1/2} m_{ij} \quad (3)$$

where, the dimensionless factor m_{ij} is obtained by interpolation from **Table 1** of values depending on relative minimal and maximal distances between loops, where the main factors m_{12} , m_{13} , m_{14} and m_{23} . The rest can be found from symmetry of the configuration.

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As for Helmholtz coil, it appears from a simple calculation that four modes of resonance are possible: A three counter-current modes, with opposite currents, corresponding to a field gradient and a co-current mode, with identical currents, corresponding to a symmetric field, appropriate for MRI. The established formula for the co-current mode resonance frequency enables us to determine the value of the tuning capacitors. On a practical level, the four coils have the same resonance frequency f_0 (Ould Ahmed Ghaly *et al.*, 2005a; Giraudeau *et al.*, 2011; Ould Ahmed Ghaly *et al.*, 2005b) higher than that of the probe f_1 . The tuning of one of the circuits can be carried out by short-circuiting the others. By predetermination of f_0 a pre-tuning of the coil can be done. Then, if necessary, the coil may be fine-tuned to resonance at the operating frequency f_1 . The coil matching was done by the use of a fifth inductively coupled ring, tuned to f_0 and with identical diameter to those of the outer rings of the coil. Thus, the coil stay electrically balanced and losses are limited.

According to the symmetry of the coils, there are in fact some obvious simplifications which are $L_4 = L_1$, $L_3 = L_2$, $C_4 = C_1$, $C_3 = C_2$, $M_{34} = M_{12}$ and $M_{24} = M_{13}$. Using Kirchoff's voltage law, the equivalent circuit equations are given in Equation (4):

$$\begin{cases} \left(j \frac{L_1}{L_2} \left(L_2 \omega - \frac{1}{C_2 \omega} \right) + j M_{14} \omega \right) I_1 + \left(j (M_{12} + M_{13}) \omega \right) I_2 = 0 \\ \left(j \left(L_2 \omega - \frac{1}{C_2 \omega} \right) + j M_{23} \omega \right) I_2 + \left(j (M_{12} + M_{13}) \omega \right) I_1 = 0 \end{cases} \quad (4)$$

Table 1. Values of the m_{ij} factors for the elliptical four coil system

m_{ij}	m_{12}	m_{13}	m_{14}	m_{23}
Elliptical coil	0.6198	0.2498	0.901	0.78951

Table 2. Values of the tuning capacitors and tuned-circuit resonance frequency

	Resonance frequency (MHz)	Tuning capacitor (pF)
Helmholtz coil	105.17	19.49
Elliptical coil	116.76	$C_1 = C_4 = 23.95$ $C_2 = C_3 = 15.37$

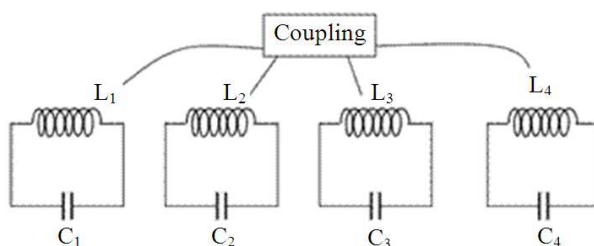


Fig. 1. Lossy equivalent circuit of the four-coil system

By analogy with Helmholtz coil, the resonance equation is shown in Equation (5):

$$\begin{cases} L_1 L_2 - (2L_1 L_2 + L_1 M_{23} + L_2 M_{14}) \frac{\omega^2}{\omega_0^2} + \\ \left[(L_1 + M_{14})(L_2 + M_{23}) - (M_{12} + M_{13})^2 \right] \frac{\omega^4}{\omega_0^4} = 0 \\ 1 - (2 + K_{14} + K_{23}) \frac{\omega^2}{\omega_0^2} + \\ \left[(1 + K_{14})(1 + K_{23}) - (K_{12} + K_{13})^2 \right] \frac{\omega^4}{\omega_0^4} = 0 \end{cases} \quad (5)$$

The resonance frequency f_I of the co-current mode, which produces a uniform field, is given by Equation (6):

$$\left(\frac{\omega_1}{\omega_0} \right)^2 = \frac{2 + K_{14} - K_{23} - \sqrt{(K_{14} - K_{23})^2 + 4(K_{12} + K_{13})^2}}{2 \left[(1 + K_{14})(1 - K_{23}) - (K_{12} + K_{13})^2 \right]} \quad (6)$$

Globally, for our system, the current ratio between the center and outer loops is given by Equation (7):

$$\left(\frac{I_1}{I_2} \right)_{\text{oi}} = \sqrt{\frac{L_2}{L_1} \cdot \frac{K_{14} - K_{23} - \sqrt{(K_{14} - K_{23})^2 + 4(K_{12} + K_{13})^2}}{2(K_{12} + K_{14})}} \quad (7)$$

Table 2 gives the values of the tuning capacitors and tuned-circuit resonance frequency, obtained from the equivalent electrical model in Equation (6).

3. B₁ FIELD CALCULATION

When performing the calculation of the magnetic field produced by the four-coil probe, the structure which consists of discrete elements of the same shape have small dimensions compared to the wavelength, thus it is possible within the framework of the quasi-static assumption to apply the superposition principle to obtain the total field. At the elementary level, the component along the y axis of the magnetic field shown in **Fig. 2** produced by a thin ring of wire, of radius a, lying in the xz plane, centered on the origin O and carrying a current I is easily determined by using Biot-Savart's law which is given in Equation (8):

$$B_{1y} = \frac{\mu_0 I}{2a(1 + [y/a]^2)^{3/2}} \quad (8)$$

where, B_{1y} is the field along the y axis, which is the axis of the coil. The extension to our case, defining four translation parameters, enables us to obtain the total field B_{1y} along the coil axis shown in Equation (9):

$$B_{1y} = \sum_{i=1}^4 \frac{\mu_0 I_i}{2a_i} \frac{1}{\left(1 + \left[\frac{y - y_i}{a_i} \right]^2 \right)^{3/2}} \quad (9)$$

where, y_i is the distance from the origin and a_i is the radius of loop i.

Table 3. Dimensions of the coils

	Center loops		Outer loops	
	Radius	Distance	Radius	Distance
Helmholtz coil	2.4 cm	2.4 cm	--	--
Elliptical coil	2.4 cm	1.38 cm	1.75 cm	3.96 cm

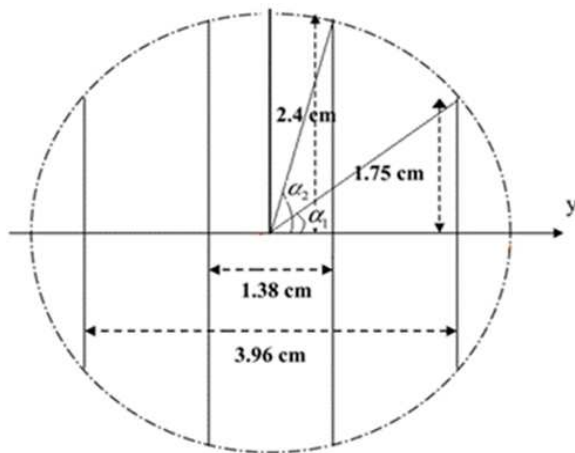


Fig. 2. Geometry of the elliptical four coils system

If one is interested in an ellipsoidal structure of given dimensions, the calculation parameters are brought back to two because of the symmetry of the structure. Thus, one can take for example the diameter of the coils or the distance between the inner and outer coils. For a partially defined geometry, by specifying one parameter, it is possible to use a computational tool to check the field homogeneity according to the second parameter. Cases of non-homogeneity of the B₁ axial field are characterized either by undulations or by a non-optimal lobe widening.

With the aim of a practical analysis of the homogeneity of the total magnetic field in the vicinity of the origin O, we can use, for a single circular loop, the formula proposed by Remeo and Hoult (1984) for the magnetic field axial component is given in Equation (10):

$$B_{1y} = \sum_{i=1}^4 (\mu \cdot I_i / 2r_i) \sin \alpha_i \left\{ \sum_{n=0}^{\infty} P_{n+1,1}(\cos \alpha_i) (y / r_i)^n \right\} \quad (10)$$

Where α_i angles are demonstrated in **Fig. 2**, $P_{n, m}(\cos \alpha_i)$ are the associated Legendre polynomials and $r_i = a_i / \sin \alpha_i$. It should be remembered that the current ratio between center and outer loops is expressed by Equation (7). **Table 3** gives the dimensions of the coils.

4. RESULTS AND DISCUSSION

After calculation, it appears that for the presented structure, starting from a spherical surface and stretching this one along the probe axis there is a certain range of possibilities for a fourth-order homogeneous field and in this last there is advantageously a single and optimal solution for a 6th order homogeneity.

The axial profile of the B₁ is calculated using the Biot-savart's law by superposition principle normalized to its maximum value shown in **Fig 3**, where in addition a comparison is done with a Helmholtz coil contained in the same reference sphere.

It is clear from **Fig. 3** that the proposed structure has a significant increase in the profile widths i.e. in considering uniformity of 1% of the maximum, the profile width has increased approximately 100% and at 10% uniformity the profile width increased 58% more than Helmholtz coil. Another interesting point is that, for identical current in the two coils, the field produced at the center for the proposed four-coil system is 1.70 times larger than that of Helmholtz coil which will increase the sensitivity of the probe.

To calculate B₁ in 3D, definition of magnetic field components for a single loop at any location can be done by using cylindrical coordinates (Kneopfel, 2000) ρ -the radial direction and y-the coil axis direction, formally called z in cylindrical coordinates but y is used here instead. Thus in the presented four-coils system, the expression for the total magnetic field is just the sum of the individual fields from each loop shown in Equation (11):

$$\left\{ \begin{aligned} B_{\rho} &= \sum_{i=1}^4 \frac{\mu I_i}{4\pi \rho} \frac{y - y_i}{\sqrt{(a_i \rho)^2 + (y - y_i)^2}} \left[-K(k) + \frac{a_i^2 + \rho^2 + (y - y_i)^2}{(a_i - \rho)^2 + (y - y_i)^2} E(k) \right] \\ B_y &= \sum_{i=1}^4 \frac{\mu I_i}{4\pi \sqrt{(a_i + \rho)^2 + (y - y_i)^2}} \left[-K(k) + \frac{(y - y_i)^2 a_i^2 - \rho^2}{(a_i - \rho)^2 + (y - y_i)^2} E(k) \right] \end{aligned} \right. \quad (11)$$

where, y_i is the distance from the origin, $E(k)$ and $K(k)$ are elliptical functions given by Equation (12):

$$K(k) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1-k^2 \sin^2 \alpha}}$$

$$E(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \alpha} d\alpha \tag{12}$$

$$k = \sqrt{\frac{4\alpha\rho}{(\alpha+\rho)^2 + \chi^2}}$$

Figure 4 below shows the contour plot of magnetic field B_1 versus the both components for the elliptical coil.

It should be noted that when the dimensions of the elliptical four coil are changed, the equivalent electric model must be changed accordingly and the pre-tuning data recalculated. Thus, we can vary the both parameters radial size (d) corresponding to center ring diameter and the wire radius (R) i.e. wire thickness. By using a computational tool, we can find several solutions for d ranges from 5 to 15 cm and R from 0.5 to 15 mm. **Figure 5** shows the computed ratio currents which clearly indicates that as the center rings diameter gets larger the currents ratio gets bad and as the wire thickness gets smaller also currents ratio gets bad, this concludes small diameters is more effective for the presented structure. **Figure 6** shows the corresponding width of lobe at 10% uniformity in terms of d . It is clear from the **Fig. 6** that as the center rings diameter and wire thickness get larger the profile width decreases. Because of the mentioned findings the presented four-coil system seem to be optimized for structures with small coils.

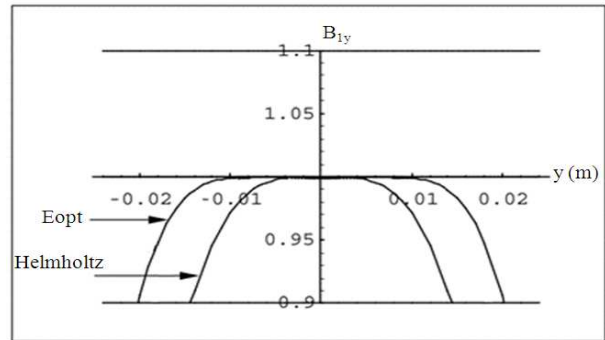


Fig. 3. Normalized profiles of B_1

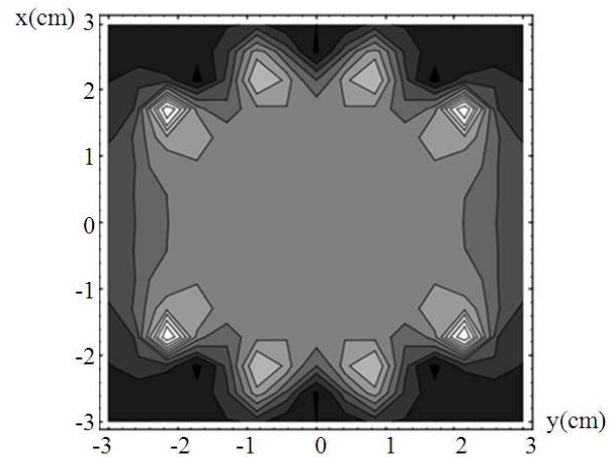


Fig. 4. Contour plot of magnetic field in the four-coil system

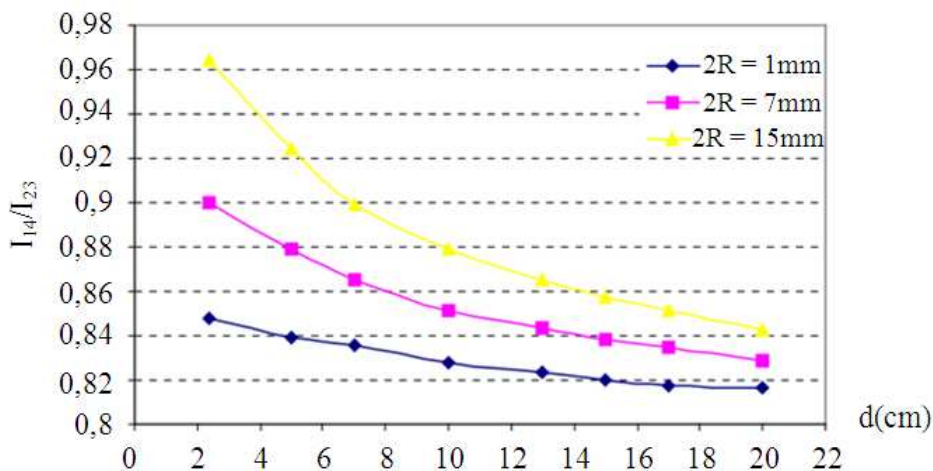


Fig. 5. Computed ratio currents in term of d

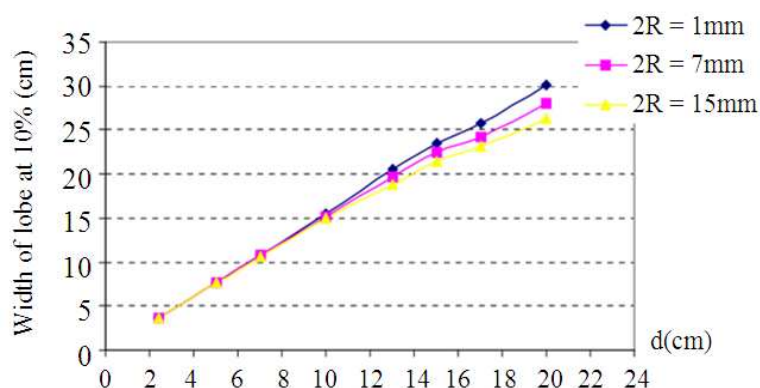


Fig. 6. Width of profile at 10% in term of d

5. CONCLUSION

A design of a free elements elliptical four-coil system that can be used for MRI is presented. It comprises four circular coaxial rings of wire in a symmetric arrangement on an ellipsoidal surface. A comparison of performances with a Helmholtz pair is presented. The developed probe has four symmetric tuned elements that consist of two different lateral coils with two tuning capacitors. A complete electrical model of the four-coil system taking into account all coupling effects is developed and enabled us to carry out the conditions of a correct operation in co-current mode. Computation of the B1 axial magnetic field was done to achieve a sixth order homogeneity. Simulations confirm the high B1 field homogeneity of the coil. Moreover a considerable improvement on field homogeneity profile width and sensitivity compared to Helmholtz coil.

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