A New High Order Algorithm with Low Computational Complexity for Electric Field Simulation

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Abstract: Problem statement: This research reported on new approach to improve speed of simulation time for free space electric wave propagation from an antenna. The existing method, Finite-Difference Time-Domain (FDTD) have been proven to solve the problem accurately, however, one of the drawbacks of the method was it needs a long processing time to simulate problem. Research efforts to increase the speed of simulating the problem are needed. **Approach:** Our recent research had found a new method with lower complexity and can simulate the problem faster than the existing FDTD algorithm. The method was developed by implementing the second order accurate discretization technique. But the method, which was named as the High Speed Low order finite-difference time-domain, had lower accuracy than the existing one. In this study, we reported on our new finding which used the $O(h^4)$ truncation error rather than $O(h^2)$ in our previous method. **Results:** The result found that we managed to recover the error and the new method still had computational complexity lower than the finite-difference time-domain. **Conclusion:** In terms of computation time, the new method also proved to solve problem faster than the conventional FDTD scheme with 9.03-63.66% reduction in computation time and also faster than the HO-FDTD with 82.48-88.99% reduction in computation time.

Key words: Numerical algorithm, finite difference method and time domain Maxwell equations

INTRODUCTION

In today's era with highly advance computer technology, numerical simulation plays a major role in the development of science and technology. The method facilitates research and industrial development in the fields. The demands of advanced wireless devices create need of tools that can facilitate research and development in the field of electromagnetic. The Finite-Difference Time-Domain (FDTD) method is one of the most credible tool in simulating electromagnetic problem, since it covers a wide range of applications (Taflove and Hagness, 2005), such as antennas, wireless and wired communication, high speed electronic circuit, biomedical and semiconductors. All of those problems are solved via Maxwell equations (Taflove and Hagness, 2005).

FDTD is a finite difference approximation to the Faraday and Ampere's laws using second order accurate in time and space. The method was first proposed by Yee (1966) used an Electric field (E), which was offset both spatially and temporally from a magnetic field grid to obtain update equations that yield the present fields throughout the computational domain in term of past fields. This method is the most commonly used to solve problem in time-domain because of its simplicity and directly adapted to homogeneous problem. Since then, the method has become one of the most powerful Maxwell equations solvers of electrodynamics.

The algorithm simplicity, robustness and potential for high complexity have attracted most researchers in computational electromagnetic field. However, there are drawbacks in the method. One of the drawbacks is it needs a long processing time to simulate problems.

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To increase the speed of FDTD, some researchers apply higher-order technique in FDTD method (Georgakopoulos *et al*., 2002; Lan *et al*., 1999; Propokidis and Tsiboukis, 2003). They developed a second order accuracy in time and fourth order in space. Result show that the higher-order method reduces the numerical dispersion and has improved stability. The implementation of higher-order truncation to Maxwell equations increase the complexity of the method, however by solving the problem in coarser grid will increase the speed of the processing time. Another approach is by taking advantages to advances in multiprocessor technology (Perlik *et al*., 1989). They apply the algorithm to connection machine.

Wave propagation in free space, is one of the problem in the propagation of electrical power between antennas (a transmitter to a receiver). This problem exist when the wave generated by a transmitter antenna propagate outward to the free space. In this research, we bound our research only up to a meter around the transmitter antenna.

In previous research, we propose a new algorithm called high speed low order FDTD algorithm (Hasan *et al*., 2005). The method has been proven to compute faster than the conventional FDTD but reduce some small amount of accuracy.

The objective of this research is to develop a fast and accurate numerical algorithm for wave propagation in free space. To achieve this objective, we used the same concept in our previous method, since the method has the "fast" characteristic that we need in this objective. Now, our problem is to "cure" the error produce by our previous method. Since, previous study shows that high order truncation can results in less error outcome, we shall use it as the medication to "heals" the approach.

In this research, we extend the method to apply $O(h^4)$ truncation instead of $O(h^2)$. The new method which is called the High Speed High Order FDTD (HSHO-FDTD) method is shown to be more potential as an alternative to FDTD method because the new method can execute faster and has higher accuracy than the conventional FDTD method.

MATERIALS AND METHODS

In order to develop the new method I discretize the transverse electric mode in one dimensional Maxwell equations time-domain form with ordinary central difference Taylor approximation for the temporal domain in both Ampere's and Faraday's law and with 3h higher order central difference Taylor approximation for the spatial domain in both Ampere's and Faraday's

law. The idea of this type of approximation is gathered from some previous research in other applications in numerical method (Sulaiman *et al*., 2004) and our previous research (Hasan *et al*., 2005). This procedure is mathematically present as follows:

Ampere's law:

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial z} \tag{1}$$

And Faraday's law:

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z} \tag{2}$$

where E_x, H_y, ε_0 and μ_0 are the electric fields, magnetic fields, electric permittivity and magnetic permeability, respectively. Discretizing (1) and (2) with the relevant approximation, gives:

$$E_x^{q+\frac{1}{2}}[m] = E_x^{q-\frac{1}{2}}[m] - \frac{\Delta t}{g\varepsilon_0\Delta z}$$
$$\left(\begin{array}{c}\frac{9}{8}\left(H_y^q\left[m+\frac{g}{2}\right]-H_y^q\left[m-\frac{g}{2}\right]\right)\\-\frac{1}{24}\left(H_y^q\left[m+\frac{3g}{2}\right]-H_y^q\left[m-\frac{3g}{2}\right]\right)\end{array}\right) \tag{3}$$

and

$$H_y^{q+1}\left[m+\frac{g}{2}\right] = H_y^q\left[m+\frac{g}{2}\right] - \frac{\Delta t}{g\mu_0\Delta z}$$
$$\left(\begin{array}{c}\frac{9}{8}\left(E_x^{q+\frac{1}{2}}[m+g]-E_x^{q+\frac{1}{2}}[m]\right)\\-\frac{1}{24}\left(E_x^{q+\frac{1}{2}}[m+2g]-E_x^{q+\frac{1}{2}}[m-g]\right)\end{array}\right) \tag{4}$$

Equation 3 and 4 with $g = 3$, will be used to solve problem in solution domain given by Fig. 1b with the black square and circle is the magnetic and electric fields have to be solved in the main HSHO(3)-FDTD algorithm. The uncalculated node, the white square and circle will be solved later only at T after the entire black node have been calculated. The standard FDTD will be executed on solution domain given by Fig. 1a.

To show the difference between the conventional FDTD, high order FDTD and the HSHO (3)-FDTD, we graphically display the computational molecule as in Fig. 2-4 with coefficient P, P1, P2, P3 and P4 are given by:

$$P = \frac{\Delta t}{\varepsilon_0 \Delta z}$$

$$P1 = \frac{1.125 \Delta t}{\varepsilon_0 \Delta z}$$

$$P2 = \frac{\Delta t}{24 \varepsilon_0 \Delta z}$$

$$P3 = \frac{0.375 \Delta t}{\varepsilon_0 \Delta z}$$

$$P4 = \frac{\Delta t}{72 \varepsilon_0 \Delta z}$$

From this Fig. 2-4, we can find that the conventional FDTD has short "hand" at both direction, but for high order FDTD and HSHO (3)-FDTD, there are also "elbows" at both directions. For high order FDTD, the distance between "elbows" is h and between "palms" is 3 h. Meanwhile, for HSHO (3)-FDTD, the distance between "elbows" is 3h and between "palms" is 9 h. For conventional FDTD, the distance between "palms" is h. There are only two same thing between all three methods are the distance between the "head" and "leg", which is t and all three methods are symmetrical in shape. "Head" and "leg" is actually the update of temporal and the difference in "hands" is the update in spatial domain.
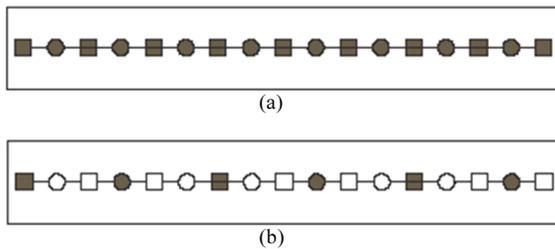

(a)

(b)

Fig. 1: (a) Solution domain for standard FDTD method and (b) Solution domain for HSHO (3)-FDTD method
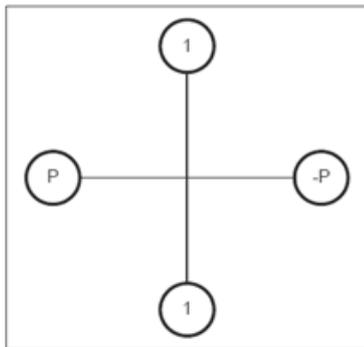

Fig. 2: Conventional FDTD computational molecule

To calculate the remaining point exist in HSHO (3)-FDTD method, we have to assume that there is no further varying time exist. Therefore, we assume at the pre-specified time step, the time varying field becomes the static field without the existing volume charge density. This assumption means that the electric field will follow the Laplace behavior. For one dimensional case:

$$\frac{\partial^2 E}{\partial x^2} = 0 \qquad (5)$$

Applying the central difference approximation to (5) yields:

$$E_{x-1} - 2E_x - E_{x+1} = 0 \qquad (6)$$

Let look at solution domain in Fig. 5.

The node points in Fig. 5, can be represents by two equation below:

$$E_1 - 2E_2 + E_3 = 0 \qquad (7)$$

and:

$$E_2 - 2E_3 + E_4 = 0 \qquad (8)$$

We can rewrite these two equations in matrix form as below:

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} -E_1 \\ -E_4 \end{bmatrix}$$
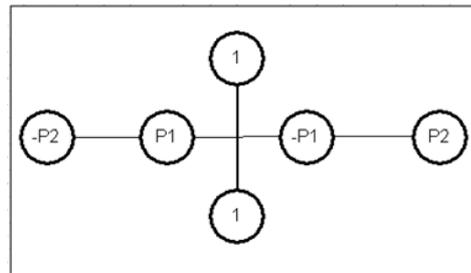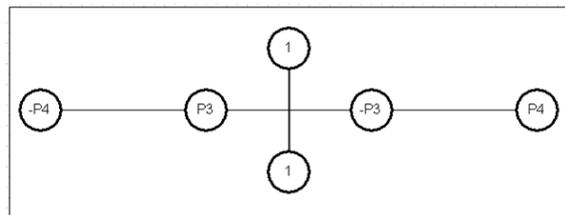

Fig. 3: High order FDTD computational molecule

Fig. 4: HSHO (3)-FDTD computational molecule

Therefore the unknown E_2 and E_3 can be calculate by the following matrix relation:

$$\begin{bmatrix} E_2 \\ E_3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -E_1 \\ -E_4 \end{bmatrix}$$

Extracting from the matrix equation above yields:

$$E_2 = \frac{1}{3}(2E_1 + E_4) \rightarrow E_2 = E_1 + \frac{1}{3}(E_4 - E_1) \tag{9}$$

and:

$$E_3 = \frac{1}{3}(E_1 + 2E_4) \rightarrow E_3 = E_1 + \frac{2}{3}(E_4 - E_1) \tag{10}$$

Equation 9 and 10 will be used for the calculating the remaining node points base on the nearest neighbor node points.

The data gathered above have been fed into worksheets to perform accuracy and wave behavior analysis on the results.

Algorithm and analysis: The ultimate objective of this study is to develop an algorithm that is not only fast in computation but also accurate in simulating problem. Using (3), (4), (9) and (10) we develop the HSHO (3)-FDTD algorithm. The HSHO (3)-FDTD algorithm can be written in either direct-domain or temporary-domain approach. In this study, we will only consider the direct-domain approach. The algorithm is as follows:

- Initialize all variable
- Setting all coefficient
- For every time steps until N_t time step
  - Update electric field with Eq. 3 for points in $0+3j$ with j from 1 until<N_p
  - Generate Gaussian pulse source at the centre of electric field
  - Update magnetic field with Eq. 4 for points in $2+3j$ with j from 1 until<N_p
- Calculate the remaining point with (9) and (10)

Fig. 5: Electric field node points after main HSHO (3)-FDTD loop

The prime differences of FDTD, high order FDTD and HSHO (3)-FDTD algorithm is the calculation for the electric and magnetic field. Assume that every calculation of electric and magnetic fields operation as O(5). This is because both update equation (equation (3) and (4)) use five point on the right hand side of equations. The calculations for electric field loop are done For N_p grid point for high order FDTD and FDTD, but $N_p/3$ for HSHO (3)-FDTD. Suppose that N_t, is the total time step in the simulation, therefore the total operation complexity is $\theta(5N_pN_t)$ for high order FDTD and total operation complexity for FDTD is $\theta(3N_pN_t)$ but $\theta(5N_pN_t/3)$ for HSHO (3)-FDTD. That is not all for HSHO (3)-FDTD. In HSHO (3)-FDTD, we also have to calculate the computational complexity for the remaining node points. From Eq. 9 and 10, the complexity is $\theta(4N_p/3)$, so the overall computational complexity for HSHO (3)-FDTD is $\theta(5N_pN_t/3+4N_p/3)$. The scenario is the same for calculation of magnetic field.

RESULTS

In this study, we execute the direct domain approach of HSHO (3)-FDTD method. The effectiveness of the method is analyzed by executing a one dimensional free space wave propagation problem with Gaussian pulse as the point source. We generate a 2.4 GHz Gaussian pulse at the middle of the solution domain of 2 m, truncated with simple absorbing boundary condition. To ensure the accuracy of the simulated result, the solution domain is discretize into 600 grid points with cell size of 0.0033 m and time slice size of 5.5×10^{-12} sec. The wave length of frequency 2.4 GHz is divided by 37 cells. The experiment was run on Intel Pentium 3 of Mobile CPU 1 GHz 727 MHz 256 MB of RAM with LINUX Operating system. The result of simulations are analyzed and summarized in Table 1 and Fig. 6.
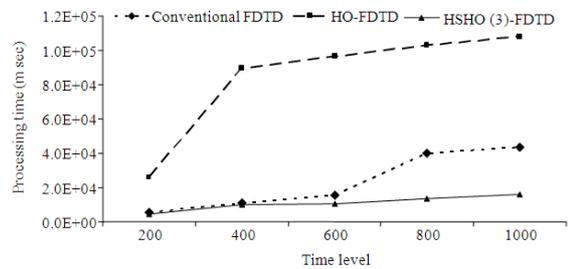

Fig. 6: Comparison of computation time for FDTD, HO-FDTD and HSHO (3)-FDTD

Table 1: Comparison of global error for HSHO (3)-FDTD and FDTD compared relatively to HO-FDTD

Time step	Method	
	HSHO (3)-FDTD	Conventional FDTD
200	0	2.38E-17
400	3.38E-11	2.03E-10
600	5.53E-13	4.14E-12
800	1.47E-12	1.52E-11
1000	4.78E-10	6.53E-10

DISCUSSION

The objective of this research is to develop a fast and accurate numerical algorithm for wave propagation in free space. To analyze whether the newly proposed algorithm achieved the objective, we analyze the accuracy of the method by comparing global error and computation time by both method and the high order FDTD method.

From Table 1, HSHO (3)-FDTD algorithms have shown very similar results to HO-FDTD (Georgakopoulos *et al*., 2002; Lan *et al*., 1999; Propokidis and Tsiboukis, 2003) compared with conventional FDTD (Yee, 1966). This means that HSHO (3)-FDTD has better accuracy in term of global errors of all simulation results. Albeit we use the second neighbor instead of the nearest neighbor that used in standard FDTD, the high order error introduce by $O(h^4)$ truncation have recover the error introduce by the jumping point method. The results are shown in power density unit. These findings fulfill the objective of developing an accurate numerical algorithm.

The comparison of computation time is given in Fig. 6. From Fig. 6, we can see that the new schemes simulate the problem faster than the conventional FDTD scheme with 9.03-63.66% reduction in computation time and also faster than the HO-FDTD with 82.48-88.99% reduction in computation time. The jumping point approach which has been recommended by Sulaiman *et al*. (2004) and Hasan *et al*. (2005) is very useful to reduce computational complexity. This successful implementation of the jumping point concept to the fourth order FDTD approach fulfills our objective of developing fast algorithm. We believe that the saving of execution time by this new method will be very significant if larger problem with higher time level is solved. From these findings we recommend to implement higher order discretization with the jumping point approach for increasing the speed of methods and accuracy.

CONCLUSION

The ultimate objective of this study was to develop a new algorithm that has lower overall computational complexity for finite-difference time-domain scheme. The low computational complexity will off course speed-up the computational time of the scheme. We also wish that an additional characteristic of the new scheme will also have high accuracy criteria.

In this study, we have developed a method call high speed high order (3)-FDTD method. The method is develop by using Taylor series approximation with $O(h^2)$ truncation in temporal domain and $O(h^4)$ truncation in spatial domain for both Ampere's and Faraday's law. However, this type of truncation will leads to higher complexity. However by solving only one-third of node points in the solution domain, overall complexity per solution domain is still reduce, i.e., less complexity than the conventional FDTD or even less than the conventional high order FDTD.

The amount of computational complexity for HSHO(3)-FDTD, $\theta(5N_pN_t/3)$ which have been calculated previously is clearly less than computational complexity for high order FDTD, $\theta(5N_pN_t)$ and computational complexity for FDTD, $\theta(3N_pN_t)$.

In this study, we also conduct some numerical experiment. Our numerical experiment show that the HSHO (3)-FDTD has the faster computational time speed and then follow by FDTD and high order FDTD. This findings is as expected since the complexity of HSHO(3)-FDTD is less complex than FDTD and High order FDTD.

From the numerical experiment, HSHO (3)-FDTD also produce a very good simulation result compare to result produce by the conventional FDTD and high order FDTD. These findings are because of the order of truncation used by each method.

The HSHO (3)-FDTD method give us the opportunity to solve 1/3 grid point of the solution domain in the main loop of HSHO (3)-FDTD and the remaining point only at the required time step. Albeit the complexity per node point increases, the overall complexity in the solution domain for HSHO (3)-FDTD is far less then the conventional FDTD. This is the reason why HSHO (3)-FDTD has better speed than FDTD algorithm. The $O(h^4)$ truncation used in HSHO (3)-FDTD also gives advantage to the new method instead of the conventional FDTD in terms of accuracy.

Performance of this scheme was tested for problem in one dimensional free space propagation with perfectly conducting boundary condition and compared to the conventional FDTD. The accuracy of both methods is then compared relatively to HO-FDTD result. The major advantages of this scheme are that it requires less processing time and has higher accuracy than the conventional FDTD. However, this new method has been used only in the problem of free space

wave propagation. We have not tried it to solve scattering problem or other more complicated problem. Up to this moment, we limit our method only to solve the free space problem with absorbing boundary and perfectly boundary condition only.

It is clearly shown that HSHO (3)-FDTD is better than conventional FDTD in one-dimension for free space wave propagating simulation.

In the future, we will apply this method to solve more complex problem and also implement the strategy used in this study to develop new methods for other area as well.

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