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Periodic Review Probabilistic Multi-Item Inventory System with Zero Lead Time under Constraint and Varying Holding Cost

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Abstract: Problem statement: This study treats the probabilistic safety stock n-items inventory system having varying holding cost and zero lead-time subject to linear constraint. **Approach:** The expected total cost is composed of three components: the average purchase cost; the expected order cost and the expected holding cost. **Results:** The policy variables for this model are the number of periods N_r^* and the optimal maximum inventory level Q_{mr}^* and the minimum expected total cost. **Conclusion/Recommendations:** We can obtain the optimal values of these policy variables by using the geometric programming approach. A special case is deduced and an illustrative numerical example is added.

Key words: Probabilistic safety stock multi-item, zero lead-time, varying holding cost, constrained probabilistic inventory system, random variable, demand fluctuations, geometric programming techniques, orthogonal conditions.

INTRODUCTION

In many situations demand is probabilistic since it is a random variable having a known probability distribution. All researchers have studied unconstrained probabilistic inventory models assuming the holding cost to be constant. Hadley and Within (1963) and Taha (1997) and Ben-Daya (1999) have examined unconstrained probabilistic inventory problems.

Fabrycky and Banks (1967) studied the probabilistic single-item, single source inventory system with zero lead-time, using the classical optimization. Also Hariri and Abou-El-Ata (1995; 1997) and Kotb (1998) investigated the constrained deterministic inventory models using a geometric programming approach. Recently, Abou-El-Ata (2002) and Fergany (2005) introduced the probabilistic multi-item inventory system with zero lead time under constraints and varying order cost, using geometric programming approach.

The aim of this study is to investigate the probabilistic safety stock multi-item, single source inventory model with zero lead-time and varying holding cost. The developed models are the probabilistic safety stock multi-item, single source inventory model with zero lead-time and varying holding cost under the expected order cost constraint and the probabilistic safety stock multi-item, single source inventory model with zero lead-time and varying holding cost under the expected varying holding cost constraint. The optimal numbers of periods N_r^*, the optimal maximum inventory levels Q_{mr}^* and the minimum expected total cost are obtained. Also a special case is deduced and an illustrative numerical example is added.

Model development: The following notations are adopted for developing our model:

c_{pr} = The purchase cost of the r^{th} item,
c_{or} = The order cost of the r^{th} item per cycle
$c_{hr}(N_r)$ = The varying holding cost of the r^{th} item per period, which takes the form

$$C_{hr}(N_r) = c_{hr} N_r^{\beta}$$

where, $c_{hr}>0$ and $\beta>0$ are constant real numbers selected to provide us the best estimation of the cost function.

$\bar{H}_r$ = The expected level of inventory held per r^{th} cycle
α = The positive value representing apart of time for safety stock
x_r = A random variable represent the demand of the r^{th} item during the cycle N_r

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$F(x_r)$ = The probability density function of the demand x_r
$E(x_r)$ = The expected value of the demand

$$x_r = \int_{x_{lr}}^{x_{ur}} x_r f(x_r) dx_r$$

where, x_{ur} and x_{lr} are the maximum value and minimum value of the demand x_r, respectively

D_r = The annual demand rate of the r^{th} item per period
$E(D_r)$ = The expected annual demand D_r
Q_{mr} = The maximum inventory level of the r^{th} item
N_r = The number of period, cycle, of the r^{th} item (a decision variable) and a review of the stock level of the r^{th} item is made every N_r period
K_o = The limitation on the expected order cost
K_h = The limitation on the expected varying holding cost
E(PC) = The expected annual purchase cost
E(HC) = The expected annual holding cost
E(OC) = The expected annual ordering cost
E(TC) = The expected total cost function

The model analysis: consider an inventory process in which a review of the stock level is made every N_r periods, r = 1, 2,…,n. An amount is ordered so that the stock level is returned to its initial position designated by: Q_{mr}, r = 1, 2,…,n. To avoid shortage during N_r periods we must maintain a safety stock absorbing demand fluctuations. Also, this is done maintaining the quantity $Q_{mr} = x_{ur}$ for any cycle N_r. Hence the resulting safety stock, $E(D_r)a$, meet the exceed demands cycle N_r. The system is represented graphically in Fig. 1.

The expected annual total cost is composed of three components: the expected purchase cost, the expected varying holding cost and the expected order cost, i.e.:

$$E(TC) = E(PC) + E(OC) + E(HC)$$

where the expected annual purchase cost is given by:

$$E(PC) = \sum_{r=1}^{n} c_{pr} E(D_r)$$

and the expected annual ordering cost is given by:

$$E(OC) = \sum_{r=1}^{n} \frac{c_{or}}{N_r}$$

and the expected annual varying holding cost is given by:

$$E(HC) = \sum_{r=1}^{n} \frac{C_{hr}(N_r)\bar{H}}{N_r}$$

where, $\bar{H}$ is the average inventory given by:

$$\bar{H} = N_r\left[Q_{mr} - \frac{E(x_r)}{2}\right]$$

Since, $E(x_r) = E(Dr)N_r$, then:

$$\bar{H} = N_r\left[Q_{mr} - \frac{E(D_r)N_r}{2}\right]$$

The Optimization of the decision variables N_r and Q_{mr} can be performed if we assume that the maximum demand during the cycle, x_{ur}, is related to the expected demand during the cycle as:

$$x_{ur} = E(x_r)g(N_r) = E(D_r)N_r g(N_r)$$

where, $g(N_r)$ is a relational function, so we get:

$$\bar{H}_r = E(D_r)N_r^2\left[\frac{2g(N_r)-1}{2}\right]$$

consider the case when $g(N_r)$ is given by:

$$g(N_r) = \frac{N_r + \alpha}{N_r}$$

Then:

$$\bar{H}_r = N_r\left[\frac{E(D_r)N_r}{2} + E(D_r)\alpha\right]$$

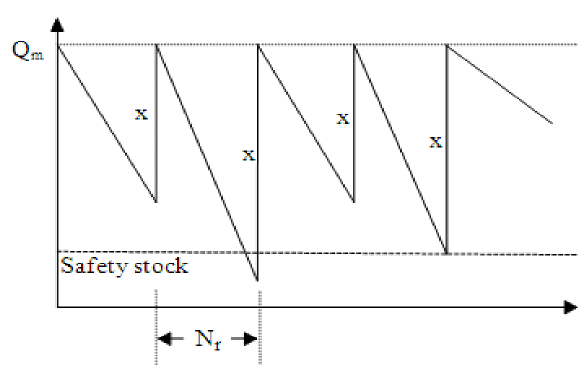

Fig. 1: Safety stock for periodic review inventory system

Then the expected varying holding cost is given by:

$$E(HC)=\sum_{r=1}^{n}\left(C_{hr}\left(N_r\right)\frac{E(D_r)N_r}{2}+c_{hr}E(D_r)\alpha\right)$$
$$=\sum_{r=1}^{n}\left(\frac{c_{hr}N_r^{\beta+1}E(D_r)}{2}+c_{hr}E(D_r)\alpha\right)$$

where, $E(D_r)a$ is the safety stock required to absorb demand fluctuations during the inventory cycle N_r.

The expected total cost is then given by:

$$E(TC)=\sum_{r=1}^{n}\left[c_{pr}E(D_r)+\frac{c_{or}}{N_r}+\frac{c_{hr}N_r^{\beta+1}E(D_r)}{2}+c_{hr}E(D_r)\alpha\right] \quad (1)$$

Our objective is to determine the optimal number of periods N_r^* that minimize the expected total cost for the following two models:

Model (I): Safety stock for Probabilistic Periodic Review Multi- Item Inventory System with Zero Lead Time and Varying Holding Cost under the Expected Order Cost Limitation

Consider the relevant expected total cost (1), the restriction on the expected order cost is:

$$\sum_{r=1}^{n}\frac{c_{or}}{N_r}\le K_o \quad (2)$$

The terms $\sum_{r=1}^{n}c_{pr}E(D_r)$ and $\sum_{r=1}^{n}c_{hr}E(D_r)\alpha$ are constants and can be postponed without any effect and then the expected total cost can be written as:

$$E(TC)=\sum_{r=1}^{n}\left[\frac{c_{or}}{N_r}+\frac{c_{hr}N_r^{\beta+1}E(D_r)}{2}\right] \quad (3)$$

Subject to:

$$\sum_{r=1}^{n}\frac{c_{or}}{N_rK_o}\le 1 \quad (4)$$

Applying the geometric programming techniques to Eqs.3 and 4, the enlarged predual function can be given by:

$$G(\underline{W})=\prod_{r=1}^{n}\left(\frac{c_{or}}{N_rW_{1r}}\right)^{W_{1r}}\left(\frac{c_{hr}N_r^{\beta+1}E(D_r)}{2W_{2r}}\right)^{W_{2r}}\left(\frac{c_{or}}{N_rk_oW_{3r}}\right)^{W_{3r}}$$
$$=\prod_{r=1}^{n}\left(\frac{c_{or}}{W_{1r}}\right)^{W_{1r}}\left(\frac{c_{hr}E(D_r)}{2W_{2r}}\right)^{W_{2r}}\left(\frac{c_{or}}{k_oW_{3r}}\right)^{W_{3r}}N_r^{W_{2r}(\beta+1)-W_{1r}-W_{3r}} \quad (5)$$

where, $\underline{W}=W_{jr}, 0<W_{jr}<1, j=1,2,3, r=1,2,....,n$ are the weights and can be chosen to yield the normal and the orthogonal conditions as follows:

$$W_{1r}+W_{2r}=1$$
$$W_{2r}\left(\beta+1\right)-W_{1r}-W_{3r}=0,\quad r=1,2,3,...,n$$

Solving the above equations, we get:

$$W_{1r}=\frac{\beta+1-W_{3r}}{\beta+2}\quad\text{and}\quad W_{2r}=\frac{1+W_{3r}}{\beta+2} \quad (6)$$

Substituting from Eq.6 into Eq.5, the dual function is given in the form:

$$g(W_{3r})=\prod_{r=1}^{n}\left(\frac{(\beta+2)c_{or}}{\beta+1-W_{3r}}\right)^{\frac{\beta+1-W_{3r}}{\beta+2}}\left(\frac{c_{hr}(\beta+2)E(D_r)}{2(1+W_{3r})}\right)^{\frac{1+W_{3r}}{\beta+2}}\left(\frac{c_{or}}{k_ow_{3r}}\right)^{W_{3r}} \quad (7)$$

Taking the logarithm of both sides of Eq. 7:

$$\ln g(W_{3r})=\sum_{r=1}^{n}\frac{\beta+1-W_{3r}}{\beta+2}\left[\ln\left(\beta+2\right)c_{or}-\ln\left(\beta+1-W_{3r}\right)\right]$$
$$+\frac{1+W_{3r}}{\beta+2}\left[\ln c_{hr}\left(\beta+2\right)E(D_r)-\ln 2\left(1+W_{3r}\right)\right]$$
$$+w_{3r}\left[\ln\frac{c_{or}}{K_o}-\ln w_{3r}\right]$$

To find the optimal value of W_{3r} which minimize $g(W_{3r})$, take the first derivative of ln $g(W_{3r})$ with respect to W_{3r} and put it equal to zero, as follows:

$$\frac{d\ln g(W_{3r})}{dW_{3r}}=\frac{-1}{\beta+2}\left[\ln\left(\beta+2\right)c_{or}-\ln\left(\beta+1-W_{3r}\right)\right]$$
$$+\frac{1}{\beta+2}\left[\ln c_{hr}\left(\beta+2\right)E(D_r)-\ln 2\left(1+W_{3r}\right)\right] \quad (8)$$
$$+\ln\frac{c_{or}}{k_o}-\ln w_{3r}-1=0$$

Simplifying Eq. 8, we obtain:

$$f(W_{3r})=\frac{c_{hr}E(D_r)(\beta+1)}{2c_{or}}\left(\frac{c_{or}}{K_oe}\right)^{\beta+2}-\frac{c_{hr}E(D_r)W_{3r}}{2c_{or}}\left(\frac{c_{or}}{K_oe}\right)^{\beta+2}$$
$$-\left(W_{3r}^{\beta+3}+W_{3r}^{\beta+3}\right)=0$$

Let:

$$A=\frac{c_{hr}E(D_r)}{2c_{or}}\left(\frac{c_{or}}{K_oe}\right)^{\beta+2}$$

Then, we obtain:

$$f(W_{3r}) = W_{3r}^{\beta+3} + W_{3r}^{\beta+2} + AW_{3r} - (\beta+1)A = 0 \quad (9)$$

Where:

$$f(0) = -(\beta+1)A < 0$$
$$f(1) = 2 - \beta A > 0$$

Which means that there exist a root $W_{3r} \in (0, 1)$. Any method such as the trial and error method could be used to calculate this root. However we shall first verify that W_{3r}^* calculated from Eq. 9 maximize $g(W_{3r})$. This is done by showing that the second derivative is always negative:

$$\frac{d^2 \ln g(W_{3r})}{dW_{3r}^{\,2}} = -\left[\frac{1}{(\beta+2)(\beta+1-W_{3r})} + \frac{1}{(\beta+2)(1+W_{3r})} + \frac{1}{w_{3r}}\right] < 0$$

Thus the root W_{3r}^* calculated from Eq.9 maximize the dual function $g(W_{3r})$. Hence the optimal solution is $W_{jr}^*, j = 1,2,3$, where W_{3r}^* is the solution of the Eq.9 and W_{1r}^*, W_{2r}^* are calculated by substituting the value of W_{3r}^* in Eq. 6.

To find the optimal number of periods N_r^*, use the following relations due to Duffin and Peterson (1974) theorem as follows:

$$\frac{c_{or}}{N_r^*} = W_{1r}^* g(W_{3r}^*)$$

$$\frac{c_{hr} N_r^{*\beta+1} E(D_r)}{2} = W_{2r}^* g(W_{3r}^*)$$

Solving these equations, the optimal number of periods per cycle is given by:

$$N_r^* = \left(\frac{2c_{or}(1+W_{3r}^*)}{c_{hr}E(D_r)(\beta+1-W_{3r}^*)}\right)^{\frac{1}{\beta+2}} \quad (10)$$

Hence the optimal maximum inventory level is given by:

$$Q_{mr}^* = E(D_r) N_r^* \left(\frac{N_r^* + \alpha}{N_r^*}\right)$$
$$= E(D_r)\left(\frac{2c_{or}(1+W_{3r}^*)}{c_{hr}E(D_r)(\beta+1-W_{3r}^*)}\right)^{\frac{1}{\beta+2}} + E(D_r)\alpha \quad (11)$$

Substituting the value of N_r^* in Eq.3 after adding the constant terms, we get:

$$\min E(TC) = \sum_{r=1}^{n}\left[\begin{array}{l} c_{pr}E(D_r) + c_{or}\left(\frac{c_{hr}E(D_r)(\beta+1-W_{3r}^*)}{2c_{or}(1+W_{3r}^*)}\right)^{\frac{1}{\beta+2}} + \\ \frac{c_{hr}E(D_r)}{2}\left(\frac{2c_{or}(1+W_{3r}^*)}{c_{hr}E(D_r)(\beta+1-W_{3r}^*)}\right)^{\frac{\beta+1}{\beta+2}} + \\ c_{hr}E(D_r)\alpha \end{array}\right] \quad (12)$$

Model (II): Safety stock for Probabilistic Periodic Review Multi- Item Inventory System with Zero Lead Time and Varying Holding Cost under the Expected Varying Holding Cost Limitation

Consider the relevant expected total cost (1), the restriction on the expected varying holding cost is:

$$\sum_{r=1}^{n} \frac{c_{hr} N_r^{\beta+1} E(D_r)}{2} \le K_h$$

The terms $\sum_{r=1}^{n} c_{pr}E(D_r)$ and $\sum_{r=1}^{n} c_{hr}E(D_r)\alpha$ are constants and can be postponed without any effect and then the expected total cost can be written as:

$$E(TC) = \sum_{r=1}^{n}\left[\frac{c_{or}}{N_r} + \frac{c_{hr}N_r^{\beta+1}E(D_r)}{2}\right] \quad (13)$$

Subject to:

$$\sum_{r=1}^{n} \frac{c_{hr} N_r^{\beta+1} E(D_r)}{2K_h} \le 1 \quad (14)$$

Applying the geometric programming techniques to Eq.13 and 14, the enlarged predual function can be given by:

$$G(\underline{W}) = \prod_{r=1}^{n}\left(\frac{c_{or}}{N_r W_{1r}}\right)^{W_{1r}}\left(\frac{c_{hr}N_r^{\beta+1}E(D_r)}{2W_{2r}}\right)^{W_{2r}}\left(\frac{c_{hr}N_r^{\beta+1}E(D_r)}{2k_h W_{3r}}\right)^{W_{3r}}$$
$$= \prod_{r=1}^{n}\left(\frac{c_{or}}{W_{1r}}\right)^{W_{1r}}\left(\frac{c_{hr}E(D_r)}{2W_{2r}}\right)^{W_{2r}}\left(\frac{c_{hr}E(D_r)}{2k_h W_{3r}}\right)^{W_{3r}} N_r^{(W_{2r}+W_{3r})(\beta+1)-W_{1r}} \quad (15)$$

where, $\underline{W} = W_{jr}, 0 < W_{jr} < 1, j = 1,2,3, r = 1,2,....,n$ are the weights and can be chosen to yield the normal and the orthogonal conditions as follows:

$$W_{1r} + W_{2r} = 1$$
$$(W_{2r} + W_{3r})(\beta + 1) - W_{1r} = 0, \qquad r = 1,2,3,...,n$$

Solving the above equations, we get:

$$W_{1r} = \frac{(\beta+1)(1+W_{3r})}{\beta+2} \text{ and } W_{2r} = \frac{1-(\beta+1)W_{3r}}{\beta+2} \qquad (16)$$

Substituting from Eq. 16 into Eq. 15, the dual function is given in the form:

$$g(W_{3r}) = \prod_{r=1}^{n}\left(\frac{(\beta+2)c_{or}}{(\beta+1)(1+W_{3r})}\right)^{\frac{(\beta+1)(1+W_{3r})}{\beta+2}}\left(\frac{c_{hr}(\beta+2)E(D_r)}{2(1-(\beta+1)W_{3r})}\right)^{\frac{1-(\beta+1)W_{3r}}{\beta+2}} \left(\frac{c_{hr}E(D_r)}{2K_h W_{3r}}\right)^{W_{3r}} \qquad (17)$$

Taking the logarithm of both sides of Eq.17:

$$\ln g(W_{3r}) = \sum_{r=1}^{n}\frac{(\beta+1)(1+W_{3r})}{\beta+2}\left[\ln(\beta+2)c_{or} - \ln(\beta+1)(1+W_{3r})\right] + \frac{1-(\beta+1)W_{3r}}{\beta+2}\left[\ln c_{hr}E(D_r)(\beta+2) - \ln 2(1-(\beta+1)W_{3r})\right] + W_{3r}\left[\ln\frac{c_{hr}E(D_r)}{2K_h} - \ln W_{3r}\right]$$

To find the optimal value of W_{3r} which minimize $g(W_{3r})$, take the first derivative of ln $g(W_{3r})$ with respect to W_{3r} and put it equal to zero, as follows:

$$\frac{d\ln g(W_{3r})}{dW_{3r}} = \frac{\beta+1}{\beta+2}\left[\ln(\beta+2)c_{or} - \ln(\beta+1)(1+W_{3r})\right] - \frac{\beta+1}{\beta+2}\left[\ln c_{hr}E(D_r)(\beta+2) - \ln 2(1-(\beta+1)W_{3r})\right] + \ln\frac{c_{hr}E(D_r)}{2K_h} - \ln W_{3r} - 1 = 0 \qquad (18)$$

Simplifying Eq. 18, we obtain:

$$f(W_{3r}) = W_{3r}^{\frac{\beta+2}{\beta+1}}(1+W_{3r}) + \frac{2c_{or}W_{3r}}{c_{hr}E(D_r)}\left(\frac{c_{hr}E(D_r)}{2K_h e}\right)^{\frac{\beta+2}{\beta+1}} - \frac{2c_{or}}{(\beta+1)c_{hr}E(D_r)}\left(\frac{c_{hr}E(D_r)}{2K_h e}\right)^{\frac{\beta+2}{\beta+1}} = 0$$

Let:

$$A = \frac{2c_{or}}{(\beta+1)c_{hr}E(D_r)}\left(\frac{c_{hr}E(D_r)}{2K_h e}\right)^{\frac{\beta+2}{\beta+1}}$$

Then, we obtain:

$$f(W_{3r}) = W_{3r}^{\frac{2\beta+3}{\beta+1}} + W_{3r}^{\frac{\beta+2}{\beta+1}} + A(\beta+1)W_{3r} - A = 0 \qquad (19)$$

Where:

$$f(0) = -A < 0$$
$$f(1) = 2 + A\beta > 0$$

Which means that there exist a root $W_{3r} \in (0, 1)$. Any method such as the trial and error method could be used to calculate this root. However we shall first verify that W_{3r}^* calculated from Eq. 19 maximize $g(W_{3r})$. This is done by showing that the second derivative is always negative:

$$\frac{d^2\ln g(W_{3r})}{dW_{3r}^2} = -\left[\frac{(\beta+1)}{(\beta+2)(1+W_{3r})} + \frac{(\beta+1)^2}{(\beta+2)(1-(\beta+1)W_{3r})} + \frac{1}{W_{3r}}\right] < 0$$

Thus the root W_{3r}^* calculated from (19) maximize the dual function $g(W_{3r})$. Hence the optimal solution is $W_{jr}^*, j = 1,2,3$, where W_{3r}^* is the solution of (19) and W_{1r}^*, W_{2r}^* are calculated by substituting the value of W_{3r}^* in Eq.16.

To find the optimal number of periods N_r^*, use the following relations due to Duffin and Peterson (1974) theorem as follows:

$$\frac{c_{or}}{N_r^*} = W_{1r}^* g(W_{3r}^*)$$
$$\frac{c_{hr}N_r^{*\beta+1}E(D_r)}{2} = W_{2r}^* g(W_{3r}^*)$$

Solving these equations, the optimal expected number of periods per cycle is given by:

$$N_r^* = \left[\frac{2c_{or}(1-(\beta+1)W_{3r})}{c_{hr}E(D_r)(\beta+1)(1+W_{3r})}\right]^{\frac{1}{\beta+2}} \qquad (20)$$

Hence the optimal maximum inventory level is given by:

$$Q_{mr}^* = E(D_r)\left[\frac{2c_{or}(1-(\beta+1)W_{3r})}{c_{hr}E(D_r)(\beta+1)(1+W_{3r})}\right]^{\frac{1}{\beta+2}} + E(D_r)\alpha \qquad (21)$$

Table 1: the parameters of the three items

Item parameters	Item 1	Item 2	Item 3
$E(D_r)$	32.00	25.00	18.00
c_{hr}	0.20	0.22	0.24
c_{or}	150.00	170.00	190.00
c_{pr}	100.00	120.00	140.00

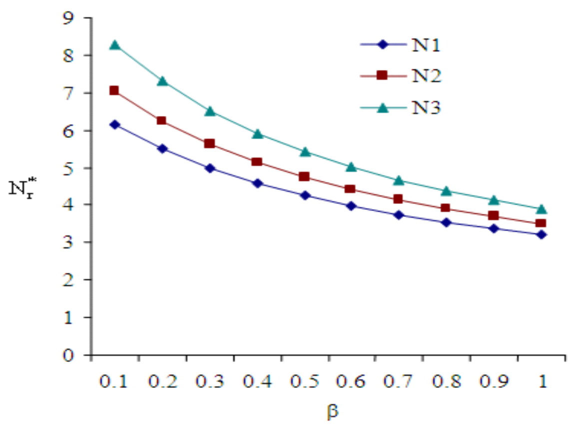

Fig. 2: The relation between N_r^* and β, $K_o = 200$

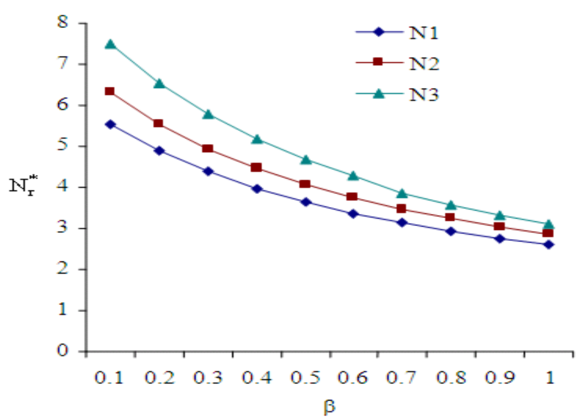

Fig. 3: The relation between N_r^* and β, $K_h = 100$

Substituting the value of N_r^* in Eq. 13 after adding the constant term, we get:

$$\min E(TC) = \sum_{r=1}^{n}\left[\begin{array}{l} c_{pr}E(D_r)+c_{or}\left[\dfrac{c_{hr}E(D_r)(\beta+1)(1+W_{3r})}{2c_{or}\left(1-(\beta+1)W_{3r}\right)}\right]^{\frac{1}{\beta+2}} \\ +\dfrac{c_{hr}E(D_r)}{2}\left[\dfrac{2c_{or}\left(1-(\beta+1)W_{3r}\right)}{c_{hr}E(D_r)(\beta+1)(1+W_{3r})}\right]^{\frac{\beta+1}{\beta+2}} \\ +c_{hr}E(D_r)\alpha \end{array}\right] \quad (22)$$

Special case: We deduce a special case of our models as follows.

For Model (I), let β = 0, r = 1 and $K_o \to \infty$ then $C_{hr}(N_r) = c_{hr}$ and $W_{3r}^* \to 0$. Also, for Model (II), let β = 0, r = 1 and $K_h \to \infty$ then $C_{hr}(N_r) = c_{hr}$ and $W_{3r}^* \to 0$. Then Eq. 10 and 20 become:

$$N^* = \sqrt{\frac{2c_o}{c_h E(D)}} \quad (23)$$

Also, Eq. 11 and 21 become:

$$Q_m^* = E(D)\sqrt{\frac{2c_o}{c_h E(D)}} + E(D)\alpha \quad (24)$$

Also, Eq. 12 and 22 become:

$$\min E(TC) = c_p E(D) + \sqrt{2c_o c_h E(D)} + c_h E(D)\alpha \quad (25)$$

This is a probabilistic single-item inventory model without any restriction and constant costs, which agree with the model of maintaining stock to absorb demand fluctions (Fabrycky and Banks, 1967)

An illustrative example: Consider the inventory parameters given in Table 1, we will find the optimal inventory doctrine by determining the minimum expected total cost when:

- The system is probabilistic periodic review multi-item inventory system under the expected order limitation $K_o = 200$
- The system is probabilistic periodic review multi-item inventory system under the expected varying holding cost limitation $K_h = 100$

Also assume that a = 5 and $0.1 \le \beta \le 1$.

Using the results of our models, the optimal expected number of periods per cycle, the optimal maximum inventory level and the minimum expected total cost min E(TC) can be summarized in the following Table 2 and 3.

The solution of the problem may be determined more readily by plotting min E(TC) against β and N_r^* against β for the two inventory models in the following Fig. 2-5.

Table 2: The optimal solution, $K_o = 200$

β	N_1^*	Q_{m1}^*	N_2^*	Q_{m2}^*	N_3^*	Q_{m3}^*	min E(TC)
0.1	6.16683	357.339	7.02215	300.554	8.29147	355.327	8941.81
0.2	5.50808	336.259	6.23161	280.790	7.29529	323.449	8954.61
0.3	4.99926	319.976	5.62184	265.546	6.52868	298.918	8967.08
0.4	4.59296	306.975	5.13629	253.407	5.92095	279.470	8979.22
0.5	4.25977	296.313	4.73954	243.489	5.42709	263.667	8991.01
0.6	3.98080	287.386	4.40869	235.217	5.01765	250.565	9002.44
0.7	3.74335	279.787	4.12825	228.206	4.67262	239.524	9013.52
0.8	3.53851	273.232	3.88732	222.183	4.37788	230.092	9024.23
0.9	3.35984	267.515	3.67800	216.950	4.12321	221.943	9034.58
1	3.20253	262.481	3.49442	212.360	3.90099	214.832	9044.56

Table 3: The optimal solution, $K_h = 100$

β	N_1^*	Q_{m1}^*	N_2^*	Q_{m2}^*	N_3^*	Q_{m3}^*	min E(TC)
0.1	5.52414	336.772	6.30517	282.629	7.48797	224.784	8942.19
0.2	4.88502	316.321	5.54002	263.500	6.52647	207.476	8955.04
0.3	4.38068	300.182	4.93893	248.473	5.77547	193.958	8967.54
0.4	3.97504	287.201	4.45747	236.437	5.17707	183.187	8979.64
0.5	3.64342	276.589	4.06533	226.633	4.69202	174.456	8991.33
0.6	3.36841	267.789	3.74125	218.531	4.29291	167.272	9002.60
0.7	3.13235	260.237	3.46725	211.706	3.87292	162.524	9013.81
0.8	2.92851	253.732	3.23432	205.673	3.57988	158.092	9024.55
0.9	2.75984	248.215	3.02800	200.510	3.33521	154.943	9034.99
1	2.61253	243.281	2.86437	196.156	3.10129	152.832	9045.11

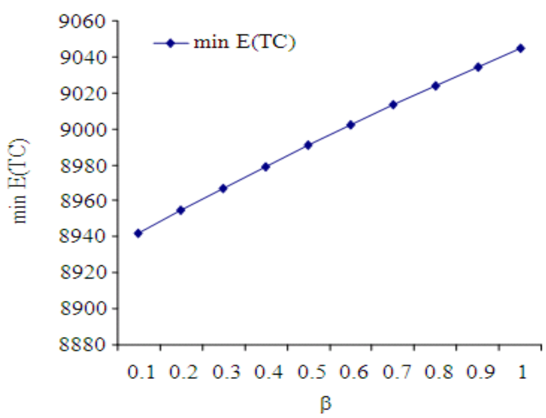

Fig. 4: The relation between min E(TC) and β, $K_o = 200$

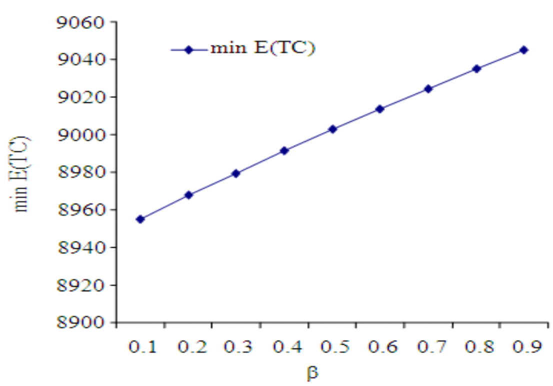

Fig. 5: The relation between min E(TC) and β, $K_h = 100$

MATERIALS AND METHODS

The aim of this study is to investigate the periodic review probabilistic multi-item inventory system with zero lead time when the holding cost is a varying function of the inventory cycle. The geometric programming approach is used to determine the optimal inventory cycle and the optimal maximum inventory level which minimize the expected total cost under the expected order cost constraint and under the expected holding cost constraint.

RESULTS AND DISCUSSION

The basic results of this chapter are.

The minimum annual expected total cost under the expected order cost constraint is given by:

$$\min E(TC) = \sum_{r=1}^{n}\left[\begin{array}{l} c_{pr}E(D_r)+c_{or}\left(\dfrac{c_{hr}E(D_r)\left(\beta+1-W_{3r}^*\right)}{2c_{or}\left(1+W_{3r}^*\right)}\right)^{\frac{1}{\beta+2}}+ \\ \dfrac{c_{hr}E(D_r)}{2}\left(\dfrac{2c_{or}\left(1+W_{3r}^*\right)}{c_{hr}E(D_r)\left(\beta+1-W_{3r}^*\right)}\right)^{\frac{\beta+1}{\beta+2}}+ \\ c_{hr}E(D_r)\alpha \end{array}\right]$$

And minimum annual expected total cost under the expected varying holding cost constraint is given by:

$$\min E(TC)=\sum_{r=1}^{n}\left[\begin{array}{l} c_{pr}E(D_r)+c_{or}\left[\dfrac{c_{hr}E(D_r)(\beta+1)(1+W_{3r})}{2c_{or}\left(1-(\beta+1)W_{3r}\right)}\right]^{\frac{1}{\beta+2}} \\ +\dfrac{c_{hr}E(D_r)}{2}\left[\dfrac{2c_{or}\left(1-(\beta+1)W_{3r}\right)}{c_{hr}E(D_r)(\beta+1)(1+W_{3r})}\right]^{\frac{\beta+1}{\beta+2}} \\ +c_{hr}E(D_r)\alpha \end{array}\right]$$

At the end of this paper, a special case of previously published work is added. Also a numerical illustrative example is added with some graphs by using Mathematica program.

CONCLUSION

We have evaluated the optimal number of periods N_r^*, r = 1, 2,…,n then we deduced the minimum expected total cost min E(TC) of the considered safety stock probabilistic multi-item inventory model. We draw the curves N_r^* and min E(TC) against β, which indicate the values of N_r^* and β that gives the minimum value of the expected total cost of our numerical example.

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