Review Articles

Neutrino Mass Mechanisms in 3-3-1 Models: A Short Review

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Abstract: In this study we review some mechanisms that provide light neutrinos in the framework of 3-3-1 gauge models without exotic leptons. In regard to the minimal 3-3-1 model, we call the attention to the fact that the perturbative regime of the model goes until 5 TeV. This requires alternative mechanisms in order to generate light neutrinos. In this review we discuss two mechanisms capable of generating light neutrinos in the framework of the minimal 3-3-1 model. In regard to the 3-3-1 model with right-handed neutrinos, we call the attention to the fact that in it mechanisms that generate light left-handed neutrinos also generate light right-handed neutrinos. Finally, we call the attention to the fact that the 3-3-1 model with right-handed neutrinos accommodate naturally the inverse seesaw mechanism.

Keywords: 3-3-1 Models, Neutrino Masses and Mixing

Introduction

Models for the electroweak interactions based on the $SU(3)_C SU(3)_L \otimes U(1)_N (3\text{-}3\text{-}1)$ symmetry were first intensively explored in the 1970's (Schechter and Ueda, 1973). However, we stress that all those models involved exotic leptons. In general, in these models exotic fermions compose the third component of the fermionic triplet. Considering the leptonic sector, we could say that there are two versions of 3-3-1 gauge models. In one version, the third component of the leptonic triplet is a simply charged lepton. The other version involves a neutral lepton as third component of the leptonic triplet.

In the 1990's it was perceived that the exotic leptons could be replaced by the standard ones. More precisely, in 1992 a 3-3-1 gauge model was proposed where the third component of the leptonic triplet was recognized as the anti-lepton (Pisano and Pleitez, 1992). In what follow, in 1993, a second version of the 3-3-1 gauge models was proposed where right-handed neutrinos was insightfully proposed as the third component of the leptonic triplet (Foot *et al.*, 1994). The first version is nowadays called the minimal version of the 3-3-1 gauge models, minimal 3-3-1 for short, because its leptonic sector is composed exclusively by the standard leptons, while the second version is called 3-3-1 model with right-handed neutrinos. In this work we review mechanisms proposed in the literature that generate light neutrinos in the framework of these two 3-3-1 gauge models.

Neutrino Mass Mechanisms in the Minimal 3-3-1 Model

Minimal here means that the leptonic sector involve the standard leptons, only. Thus, each family of leptons come inside a triplet:

$$f l_L^T = (V_l, e_l, e_l^c)_l^T \sim (1,3,0) \tag{1}$$

where, $l = e, \mu, \tau$ and T means transposition. Then, we can immediately conclude that the model may explain electric charged quantization a la Grande unification theories as showed in (Pires and Ravinez, 1998).

In addition to the standard gauge bosons, the model has five new ones, namely a new Z', two simply charged gauge bosons, $V^{\pm}$ and two doubly charged gauge bosons $U^{\pm\pm}$. We also call the attention to the fact that these four charged gauge bosons carry two units of lepton number each, called bilepton gauge bosons. We stress that the doubly charged gauge bosons constitute genuine and distinguishable signatures of the model (Cuypers and Davidson, 1998).

Concerning the scalar sector of minimal 3-3-1 model, the most complex scenario involve three triplets and one scalar sextet:

$$\eta^T = (\eta^0, \eta^-, \eta^+)^T, S = \begin{pmatrix} \sigma_1^0 & \frac{h_2^-}{\sqrt{2}} & \frac{h_2^+}{\sqrt{2}} \\ \frac{h_2^-}{\sqrt{2}} & H_1^{--} & \frac{\sigma_2^0}{\sqrt{2}} \\ \frac{h_1^+}{\sqrt{2}} & \frac{\sigma_2^0}{\sqrt{2}} & H_2^{++} \end{pmatrix} \tag{2}$$

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where these scalars have the following transformation properties under the gauge group $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$, $X \sim (1,3,-1)$, $\rho \sim (1,3,1)$, $\eta \sim (1,3,0)$ and $S \sim (1,6,0)$. The neutral components of the triplets develop VEV's, $\langle \eta^0 \rangle = \mu_\eta$ $\langle \rho^0 \rangle = \upsilon_\rho$ $\langle X^0 \rangle = \upsilon_x$, so that we have the correct pattern of symmetry breaking, with υ_x being responsible for the $SU(3)_L \otimes U_N(1)$ breaking to $SU(2)_L \otimes U(1)_Y$ symmetry, while υ_η and υ_ρ combine to break $SU(2)_L \otimes U(1)_Y$ to QED, $U(1)e$.

Considering only one scalar sextet, S, the masses of all leptons, including neutrinos, arise from a common set of Yukawa couplings:

$$G \bar{f}^c_l s^* f_l \tag{3}$$

The sextet S has two neutral components, σ^0_1 and σ^0_2 and when both develop nonzero VEV's, $\upsilon_{\sigma 1}$ and $\upsilon_{\sigma 2}$, neutrinos and charged leptons get their masses given by the following expressions, $m_\nu = G\upsilon_{\sigma 1}$ and $m_l = G\upsilon_{\sigma 2}$.

Perceive that a unique scalar sextet is not enough to generate the correct masses of all leptons of the model. The reason is simple, notice that the texture of m_ν is the same as the m_l because both involve the same matrix G. Automatically, the rotating mixing matrix, U, that diagonalizes m_ν will diagonalize m_l too. Thus, the lepton mass eigenstates, $\hat{l}$, get related to the symmetry eigenstates, l, through the following rotation, $\hat{V}l_L = Ul_a Va_L$ and $\hat{e}l_L = Ul_a e_{a_L}$, where a = 1, 2, 3. Consequently, the charged current will be always diagonal, $\frac{g}{\sqrt{2}} \bar{V}L\gamma^\mu \hat{e}_L W^+_\mu \rightarrow \frac{g}{\sqrt{2}} \bar{V}L\gamma^\mu \hat{e}_L W^+_\mu$, which goes against the recent at-mospheric and solar neutrino oscillation experiments. As an immediate consequence, three scalar triplets and one scalar sextet are not suffcient to explain the masses of all fermions of the model. This observation discards the mechanism developed in (Montero *et al.*, 2001). There are, in the 3 literature, many suggestions for solving this problem but all of them require the enlargement of the particle content of the model (Okamoto and Yasue, 1999).

On the other hand, an interesting predictions of the minimal 3-3-1 model is the one concerning the Weinberg mixing angle, θ_W, which arises from the kinetic term of the gauge bosons and is expressed by the following relation:

$$\frac{g'}{g} = \frac{S_w}{\sqrt{1-4S^2_w}} \tag{4}$$

where, $S_W = \sin \theta_W$, g is the $SU(3)_L$ coupling and g' is the $U(1)_N$ coupling. This relation shows us that a Landau pole exists for the theory and, in order to keep the theory

inside the perturbative regime, a bound on this mixing angle is obtained, $S^2_W < 0.25$. Translating this in terms of an energy scale, it was pointed out in (Dias *et al.*, 2005) that the perturbative regime of the model persists until about few TeV's. In other words, the highest energy scale where the model is perturbatively trustable is about 4-5 TeV and it can certainly be regarded as an effective model so that above that scale the underlying fundamental theory has to be called in.

In what concern neutrino masses, this is particularly worrying because, once the pertur-bative scale is around 4-5 TeV, then the effective dimension five operator:

$$\frac{h}{\Lambda} \left(\bar{f}^C_L \eta * \right) (\eta^\dagger f_L) \tag{5}$$

Generates the following mass formula to the neutrinos, $m_\nu = \frac{h\upsilon^2_\eta}{\Lambda}$, then, for $\upsilon_\eta \approx 10^2 \text{GeV}$ and $\Lambda = 5\text{TeV}$, we get $m_\nu = 10h\text{GeV}$. In other words, even the effective dimension-5 operator favors heavy neutrinos. So, in what concern neutrinos, this is the present undesirable status of the minimal 3-3-1 model. This means, too, that all proposals of generating neutrino masses through extension of the minimal 3-3-1 model that did not provide a way out to circumvent the effective dimension-5 operator can not be seeing as a realistic small neutrino mass scenario.

Concerning the problem of small neutrino masses in the minimal 3-3-1 model, as far as I know there are two ways out. The first one make use of Z_n discrete symmetries to avoid dominant effective operators of low order. For example, for a particular representation of Z_3 we can have effective dimension-11 as dominant effective operators which leads naturally to neutrino at eV scale for $\Lambda = 4$ -5TeV (Queiroz *et al.*, 2010). The second possibility requires the addition of right-handed neutrinos in the singlet form to the leptonic particle content of the minimal 3-3-1 models and then combine type I with type II seesaw mechanisms to generate tiny neutrino masses (Caetano *et al.*, 2012).

Concerning the first case, on considering the Z_3 discrete symmetry with the following representation of the fields:

$$\eta \rightarrow e^{4i\pi}\eta, \rho \rightarrow e^{-4i\pi/3}\rho, X \rightarrow e^{2i\pi/3}X, fl_L \rightarrow e^{2i\pi/3} fl_L \tag{6}$$

It is amazing that such Z_3 symmetry allows, as the first dominant effective operator that generates neutrino mass, exactly a dimension-11 operator:

$$\frac{h^\nu}{\Lambda^7} \left(\bar{f}^C_L \eta * \right) (\rho X\eta)^2 (\eta^\dagger f_L) \tag{7}$$

In this equation the term $(\rho X\eta)$ is the anti-symmetric singlet combination under $SU_L(3)$.

When η, ρ and X develop their respective VEV's, this operator generates the following expression for the neutrino masses, $m_\nu = \frac{h_\nu}{\Lambda^7} \upsilon_\eta^4 \upsilon_\rho^2 \upsilon_X^2$. That is a striking result if one observes that even for Λ around υ_x, the operator above is suppressed enough to generate small masses for the neutrino. For instance, in being conservative and taking $\upsilon_\eta \approx 10^2 \text{GeV}$, $\upsilon_\rho \approx 10 \text{GeV}$, as before, $\upsilon_x \approx 1 \text{TeV}$ and $\Lambda = 5 \text{TeV}$, we obtain a prediction for the neutrino masses, $m_\upsilon \approx 0.1 h^\nu \text{eV}$, meaning that a Λ around few TeV's is perfectly compatible with neutrino masses at the sub-eV scale, which represents an astonishing achievement for the model.

Concerning the second case, it was showed in (Ferreira *et al.*, 2011) that the minimal 3-3-1 model may be implemented with two scalar triplets, only, namely, X and ρ. This is the reduced version of the 3-3-1 model (Dong and Si, 2014). In the latter charged leptons and some quarks gain masses through effective dimension-5 operators. This is quite possible because $\Lambda = 4$-5TeV. In order to obtain small neutrino masses in this version, we add right-handed neutrinos in the singlet form and a third scalar triplet $\phi = (\phi^0 \phi_1^- \phi_2^+)^T \sim (1,3,0)$ to the other two X and ρ with the main supposition being that ϕ is heavier than the other with its mass being around Λ (Caetano *et al.*, 2012).

In this point, it is imperative to impose a Z_4 symmetry with the following fields trans-forming as:

$$\rho \to w^3 \rho, X \to w^3 X, \phi \to w^2 \phi, v_{aR} \to w^0 V_{aR}, f_{aL} \to w^2 f_{aL} \quad (8)$$

where, $w = e^{i\frac{\pi}{2}}$.

In view of this, neutrino mass terms can only arise from the Yukawa interactions below:

$$L = Y \bar{f}_L \phi V_R + \frac{M}{2} \bar{V}_R^C V_R + H.c \quad (9)$$

where, Y and M are 3×3 matrices.

All neutral fields ρ^0, X^0, ϕ^0 are assumed to develop VEV according to:

$$\langle \rho^0 \rangle = \frac{\upsilon_\rho}{\sqrt{2}}, \langle X' \rangle = \frac{\upsilon_{x'}}{\sqrt{2}} \langle \phi^0 \rangle \frac{\upsilon_\phi}{\sqrt{2}} \quad (10)$$

When ϕ develop VEV different from zero, neutrinos develop Dirac mass terms and we Have:

$$L_{mass} = \frac{Y \upsilon_\phi}{\sqrt{2}} \bar{V}_L V | R + \frac{M}{2} \bar{V}_R^C V_R + H.c \quad (11)$$

On considering the basis $V_L V_R^C$, we can write the mass terms above in the form:

$$L_{mass} = \frac{1}{2} \bar{V}^C M_\nu \nu + H.c \quad (12)$$

Where:

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M \end{pmatrix} \quad (13)$$

With $M_D = \frac{Y_{\upsilon\phi}}{\sqrt{2}}$ and M. When all values of M are larger than all elements of M_D, we obtain, after diagonalizing this mass matrix, the following expressions for the left and right-handed neutrino masses:

$$m_{\nu I} \approx M_D^T M^{-I} M_D, m_{\nu R} \approx M \quad (14)$$

This is the canonical type I seesaw mechanism (Gell-Mann *et al.*, 1978). As usual we consider M diagonal and degenerate.

In regard to the Left-Handed (LH) neutrinos, according to Equation 14 the order of magnitude of their masses is:

$$m_{\nu I} \approx \frac{\upsilon_\phi^2}{M} \quad (15)$$

As it is known, the maximum value M can acquire is around 5 TeV (Dias *et al.*, 2005). Thus for we have LH neutrinos at the eV scale, we need $\upsilon_\phi \approx 10^{-2} \text{GeV}$. There is no lower bound on this parameter. The upper bound, $\upsilon_\phi^2 + \upsilon_\rho^2 = (246)^2 \text{GeV}$, arises because υ_ρ as well as υ_ϕ both contribute to the mass of the standard charged gauge boson $W^\pm$. Thus there is no problem in taking υ_ϕ as small as we wish since we have $\upsilon_\rho \approx 246 \text{GeV}$.

A suppressed υ_ϕ can be obtained through a kind of type II seesaw mechanism (Magg, 1980) as developed in (Grimus *et al.*, 2009). For this we consider the most complete potential gauge invariant under 3-3-1 and that conserve Z_4:

$$\begin{aligned} V(\phi, \rho, X, \sigma) &= \mu_\phi^2 \phi^\dagger \phi + \mu_\rho^2 \rho^\dagger \rho + \mu_X^2 X^\dagger X \\ &+ \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (X^\dagger X)^2 + \\ &+ \lambda_4 (\phi^\dagger \phi)(\rho^\dagger \rho) + \lambda_5 (\phi^\dagger \phi)(X^\dagger X) + \lambda_6 (\rho^\dagger \rho)(X^\dagger X) \\ &+ \lambda_7 (\rho^\dagger \phi)(\phi^\dagger \rho) + \lambda_8 (X^\dagger \phi)(\phi^\dagger X) + \lambda_9 (\rho^\dagger X)(X^\dagger \rho) \\ &- \frac{f}{\sqrt{2}} \in^{ijk} \phi_i \rho_j X_k + H.c \end{aligned} \quad (16)$$

where, f is a free parameter with dimension of mass.

From this potential, we obtain the following minimum condition to ϕ develop vev:

$$\upsilon_\phi (\mu_\phi^2 + \lambda_1 \upsilon_\phi^2 + \frac{1}{2} \lambda_4 \upsilon_\rho^2 + \frac{1}{2} \lambda_5 \upsilon_X^2) - \frac{1}{2} f \upsilon_\rho \upsilon_X = 0 \quad (17)$$

The highest energy scale the model support is around 4-5TeV. We assume that the scalar ϕ belong to this energy scale. This means that $\mu_\phi \approx M$. Note that μ_ϕ is dominant in the parenthesis in the first relation of the Equation 17. As a result, we obtain from it:

$$\upsilon_\phi = \frac{f\upsilon_\rho\upsilon_X}{2M2}$$ (18)

On substituting this expression for υ_ϕ in Equation 14, we obtain the following expression to the LH neutrino mass:

$$m_\nu = \frac{\sqrt{2}}{8}Y^T Y \frac{f^2\upsilon_\rho^2\upsilon_X^2}{M^5}$$ (19)

This is the main point of our work. According to this expression neutrino masses get suppressed by high scale M^5 in its denominator. This allows we have neutrino masses at eV scale for M around few TeV's. For we see this note that for typical values $f = 1GeV$, $\upsilon_\rho = 246GeV$, $\upsilon_x = 10^3GeV$ and $M = 5 \times 10^3GeV$, we obtain:

$$m_\nu = 3.4Y^T Y eV$$ (20)

Which falls exactly at the eV scale. Whatever comes to be the texture of the neutrino masses, it can be obtained by choosing an adequate values for the elements of the matrix Y.

Neutrino Mass Mechanisms in the 3-3-1 Model With Right-Handed Neutrinos

The model we consider is the 3-3-1 model with right-handed neutrinos (Foot et al., 1994). It is one of the possible models allowed by the $SU(3)_C \otimes SU(3)_L \otimes U(1)$ gauge symmetry where leptons are distributed in the following representation content:

$$L_a \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ (\nu_{aR})^c \end{pmatrix} \sim (1,3,-1/3), e_{aR} \sim (1,1-1)$$ (21)

where, a = 1, 2, 3 refers to the three generations. After the spontaneous breaking of the 3-3-1 symmetry to the standard one, the leptonic triplets above splits into the standard leptonic doublet $L_a = (\nu_{aL}, e_{aL})^T$ plus the singlet $(\nu_{aR})^C$. Thus this model recovers the standard model with right-handed neutrinos.

We remember here that it is not a trivial task to generate light right-handed neutrinos in any simple extension of the standard model. However, in the 3-3-1 model in question, right-handed neutrinos can naturally obtain small masses through effective dimension-five operators. This is due, in part, to the fact that, in the model, the right-handed neutrinos compose, with the left-handed ones, the same leptonic triplet L. As we will

see here, it is this remarkable feature that turns feasible the raise of light right-handed neutrinos.

The gauge sector of the model is composed by the standard gauge bosons and others five gauge bosons called $V^\pm$, U^0, $U^{0\dagger}$ and Z^0. Also, the model possesses three scalar triplets, two of them transforming as, $\eta \sim (1, 3, -1 = 3)$ and $X \sim (1, 3, -1/3)$ and the other as, $\rho \sim (1, 3, 2/3)$, with the following vacuum structure:

$$\langle\eta\rangle_0 = \begin{pmatrix} \frac{\upsilon_\eta}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix}, \langle\rho\rangle_0 = \begin{pmatrix} 0 \\ \frac{\upsilon_\rho}{\sqrt{2}} \\ 0 \end{pmatrix}, \langle X\rangle_0 = \begin{pmatrix} 0 \\ 0 \\ \frac{\upsilon_X}{\sqrt{2}} \end{pmatrix}$$ (22)

These scalars are sufficient to engender spontaneous symmetry breaking and generate the correct masses for all massive particles.

In order to have the minimal model, we assume the following discrete symmetry trans-formation over the full Lagrangian:

$$(X, \eta, \rho, e_{aR}) \rightarrow -(X, \eta, \rho, e_{aR})$$ (23)

where, a = 1, 2, 3 and i = 1, 2. This discrete symmetry helps in avoiding unwanted Dirac mass term for the neutrinos (Foot et al., 1994) and implies a realistic minimal potential (Pal, 1995).

In the original version of the model, all massive particles gain masses, except neutrinos. Thus we construct all possible effective dimension-five operators in the 3-3-1 model with right-handed neutrinos that lead to neutrino masses (Alex et al., 2005). The first one involves the leptonic triplets L and the scalar triplet η. With these triplets we can form the following effective dimension-five operator:

$$LM_L = \frac{f_{ab}}{\Lambda}\left(\bar{L}_a^C \eta*\right)(\eta^\dagger L_b) + H.c$$ (24)

According to this operator, when η^0 develops a VEV, υ_η, the left-handed neutrinos develop Majorana mass terms with the form:

$$(M_L)_{ab} = \frac{f_{ab}\upsilon_\eta^2}{\Lambda}$$ (25)

Due to the fact that right-handed neutrinos are not singlets in the model in question, a second effective dimension-five operator generating neutrino masses is possible. It is con-structed with the scalar triplet X and the leptonic triplet L:

$$l_{M_R} = \frac{h_{ab}}{\Lambda}\left(\bar{L}_a^C X*\right)(X^\dagger L_b) + H.c$$ (26)

When X'^0 develops a VEV, $\upsilon_{x'}$, this effective operator provides Majorana masses for the right-handed neutrinos:

$$(M_R)_{ab} \overline{(V_{aR})}^C V_{br}, with (M_R)_{ab} = \frac{h_{ab}\upsilon_{x'}^2}{\Lambda} \quad (27)$$

Thus, we have Majorana mass terms for the neutrinos both having the same origin, i.e., effective dimension-five operators (Dias *et al.*, 2010). At this point, two comments are in order. First, as the VEV $\upsilon_{x'}$ is responsible for the breaking of the 3-3-1 symmetry to the standard symmetry and that v_η contributes to the spontaneous breaking of the standard symmetry, thus it is natural to expect that $\upsilon_{x'} > \upsilon_\eta$, which implies $M_R > M_L$. For example. Taking $\Lambda = 10^{12}$GeV, $\upsilon_\eta = 40$GeV and $\upsilon_{x'} = 5$TeV, we obtain $(M_L)_{ab} = 1.6f_{ab}$eV and $(M_R)_{ab} = 25h_{ab}$keV.

Thus, in the 3-3-1 model with right-handed neutrinos, we have left-handed neutinos with mass at eV scale and right-handed neutrinos with mass at keV scale. Those right-handed neutrinos are natural candidates for warm dark matter.

These effective operators may be realized through the addition of at least one scalar sextet (Cogollo *et al.*, 2009):

$$S = \frac{1}{\sqrt{2}}\begin{pmatrix} \Delta^0 & \Delta^- & \phi^0 \\ \Delta^- & \Delta^{--} & \phi^- \\ \phi^0 & \phi^- & \sigma^0 \end{pmatrix} \quad (28)$$

With the leptonic triplet L and the sextet S we form the Yukawa coupling $\overline{L}SL^C$. The sextet S has three neutral scalars that may develop VEV. The VEV's of the neutral scalars Δ^0 and σ^0 will lead to Majorana mass terms for both the left-handed and right-handed neutrinos respectively, while the VEV of the neutral scalar ϕ^0 will generate a Dirac mass term which mix the left-handed neutrinos with the right-handed ones. In order to implement the type II seesaw mechanism the Dirac mass terms must be avoided. For this we assume that ϕ^0 does not develop VEV and impose the set of discrete symmetries $(X, \rho, e_{aR}) \rightarrow -(X, \rho, e_{aR})$. The discrete symmetry also helps in avoiding avor changing neutral currents involving quarks and scalars and is important to obtain a simple potential. In regard to the Dirac mass terms, we think important to emphasize that, in avoiding them, the left-handed neutrinos get decoupled of the right-handed ones. Thus, in this case, we could say that our right-handed neutrinos are, in fact, completely steriles. For this reason, from now on we refer to these neutrinos as the sterile neutrinos.

The mechanism arises in the potential of the model and is communicated to the neutri-nos through the Yukawa interaction $\overline{L}L^CS$. The essence of the mechanism is that lepton number is violated explicitly through some terms in the potential. For this it is neces-sary to know the lepton number distribution of the scalars:

$L(\eta'^0, \sigma^0, \rho'^+) = -2$ and $L (X^0, X^-, \Delta^0, \Delta^-, \Delta^{--}) = 2$. When Δ^0 and σ^0 develop VEV's, automatically the left-handed neutrinos (v_L) and the sterile ones (v_R) both develop Majorana mass terms. The masses of v_L and of v_R get proportional to υ_Δ and υ_σ, respectively. The role of the type II seesaw mechanism is to furnish tiny values for υ_Δ and υ_σ.

The most complete part of the potential that obeys the discrete symmetry discussed above and conserves lepton number is composed by the following terms:

$$\begin{aligned}
V &= \mu_x^2 X^2 + \mu_\eta^2 \eta^2 + \mu_\rho^2 \rho^2 + \lambda_1 X_4 \\
&+ \lambda_2 \eta^4 + \lambda_3 \rho^4 + \lambda_4 (X^\dagger X)(\eta^\dagger \eta) \\
&+ \lambda_5 (X^\dagger X)(\rho^\dagger \rho) + \lambda_6 (\eta^\dagger \eta)(\rho^\dagger \rho) \\
&+ \lambda_7 (X^\dagger \eta)(\eta^\dagger X) + \lambda_8 (X^\dagger \rho)(\rho^\dagger X) \\
&+ \lambda_9 (\eta^\dagger \rho)(\rho^\dagger \eta) + (\frac{f}{\sqrt{2}} \epsilon^{ijk} \eta_i \rho_j X_k \\
&+ H.C) + \mu_S^2 Tr(S^\dagger S) \\
&+ \lambda_{10} Tr(S^\dagger S)^2 + \lambda_{11} [Tr(S^\dagger S)]^2 \\
&+ (\lambda_{12}\eta^\dagger\eta + \lambda_{13}\rho^\dagger\rho + \lambda_{14}X^\dagger X)_{10} Tr(S^\dagger S) \\
&\lambda_{15}(\epsilon^{ijk}\epsilon^{lmn} \rho_n \rho_k S_{li} S_{mj} + H.C) \\
&+ \lambda_{16}(X^\dagger S)(S^\dagger X) + \lambda_{17}(\eta^\dagger S)(S^\dagger \eta) \\
&+ \lambda_{18}(\rho^\dagger S)(S^\dagger \rho)
\end{aligned} \quad (29)$$

While the other part that violates explicitly the lepton number is composed by these terms:

$$\begin{aligned}
V' &= \lambda_{19}(\eta^\dagger X)(\eta^\dagger X) + \left(\frac{\lambda_{20}}{\sqrt{2}} \epsilon^{ijk} \eta_m^* S_{mi} X_j \rho k + H.C\right) \\
&+ \left(\frac{\lambda_{21}}{\sqrt{2}} \epsilon^{ijk} X_m^* S_{mi} \eta_j \rho k + H.C\right) \\
&- M_1 \eta^T S^\dagger \eta - M_2 X^T S^\dagger X
\end{aligned} \quad (30)$$

We think we have presented all the aspects of the model that are relevant to the implemen-tation of the type II seesaw mechanism, which we do next. From the Yukawa interaction:

$$L_\nu^Y = G_{ab} \overline{L}_{aL} Sl_{bL}^C + H.c \quad (31)$$

When Δ^0 and σ^0 both develop VEV, the left-handed and the sterile neutrinos develop the following mass terms:

$$L_\nu^Y = G_{ab}\upsilon_\Delta \overline{V}_{aL}^C V_{bL} + G_{ab}\upsilon_\sigma \overline{V}_{aR}^C V_{bR} \quad (32)$$

We emphasize here that the mass terms of both neutrinos have as common origin the Yukawa interaction in Equation 31. In practical terms this means that the same Yukawa couplings G_{ab} are common for the left-handed and sterile neutrino masses. That's a very interesting result because when the masses of the left-handed neutrinos get measured directly, automatically the masses of the sterile neutrinos will be predicted.

The role of the type II seesaw mechanism here is to provide tiny values for υ_Δ and υ_σ. This is achieved from the minimum condition of the potential $V + V'$. For this it is necessary to select which of the eight neutral scalars of the model develop VEV. We already discussed why ϕ^0 is not allowed here to develop VEV. In the original version of the model only η^0, ρ^0 and $X^{'0}$ were allowed to develop VEV. The reason for this is to avoid avor changing neutral currents involving quarks and scalars. Thus we have that the simplest case is when only $X^{'0}$, ρ^0, η^0, Δ^0, σ^0 develop VEV. We assume this and shift these fields in the following way:

$$X^{'0}, \rho^0, \eta^0, \Delta^0, \sigma^0 \rightarrow \frac{1}{\sqrt{2}}(\upsilon_{X',\rho,\eta,\Delta,\sigma} + R_{X',\rho,\eta,\Delta,\sigma} + il_{X',\rho,\eta,\Delta,\sigma}) \quad (33)$$

Considering the shift of the neutral scalars in Equation 33, the composite potential, $V+V'$, provides the following set of minimum conditions:

$$\mu_\rho^2 + \lambda_2\upsilon_\rho^2 + \frac{\lambda_5}{2}\upsilon_{x'}^2 + \frac{\lambda_6}{2}\upsilon_\eta^2 + \frac{f\upsilon_\eta\upsilon_{x'}}{2\upsilon_\rho} + \frac{\lambda_{13}}{4}(\upsilon_\sigma^2 + \upsilon_\Delta^2) +$$

$$\lambda_{15}\upsilon_\Delta\upsilon_\sigma - \frac{\lambda_{20}}{4}\left(\frac{\upsilon_\eta\upsilon_\Delta\upsilon_{x'}}{\upsilon_\rho}\right) + \frac{\lambda_{21}}{4}\left(\frac{\upsilon_x\upsilon_\eta\upsilon_\sigma}{\upsilon_\rho}\right) = 0$$

$$\mu_\eta^2 + \lambda_2\upsilon_\eta^2 + \frac{\lambda_4}{2}\upsilon_{x'}^2 + \frac{\lambda_6}{2}\upsilon_\rho^2 + \frac{f\upsilon_x\upsilon_\rho}{2\upsilon_\eta} + \frac{\lambda_{12}}{4}(\upsilon_\sigma^2 + \upsilon_\Delta^2) +$$

$$\frac{\lambda_{17}}{4}\upsilon_\Delta^2 - M_1\upsilon_\Delta - \frac{\lambda_{20}}{4}\left(\frac{\upsilon_\Delta\upsilon_\rho\upsilon_{x'}}{\upsilon_\eta}\right) + \frac{\lambda_{21}}{4}\left(\frac{\upsilon_x\upsilon_\rho\upsilon_\sigma}{\upsilon_\eta}\right) = 0$$

$$\mu_x^2 + \lambda_1\upsilon_{x'}^2 + \frac{\lambda_4}{2}\upsilon_\eta^2 + \frac{\lambda_5}{2}\upsilon_\rho^2 + \frac{f\upsilon_\eta\upsilon_\rho}{2\upsilon_{x'}} + \frac{\lambda_{14}}{4}(\upsilon_\Delta^2 + \upsilon_\sigma^2) +$$

$$\frac{\lambda_{16}}{4}\upsilon_\sigma^2 - M_2\upsilon_\sigma - \frac{\lambda_{20}}{4}\left(\frac{\upsilon_\eta\upsilon_\rho\upsilon_\Delta}{\upsilon_{x'}}\right) + \frac{\lambda_{21}}{4}\left(\frac{\upsilon_\eta\upsilon_\rho\upsilon_\Delta}{\upsilon_{x'}}\right) = 0$$

$$\mu_S^2 + \frac{\lambda_{10}}{2}\upsilon_\Delta^2 + \frac{\lambda_{11}}{2}(\upsilon_\sigma^2 + \upsilon_\Delta^2) + \frac{\lambda_{12}}{2}\upsilon_\eta^2$$

$$+ \frac{\lambda_{13}}{2}\upsilon_\rho^2 + \frac{\lambda_{14}}{2}\upsilon_{x'}^2 + \lambda_{15}\frac{\upsilon_\eta^2\upsilon_\sigma}{\upsilon_\Delta} +$$

$$\frac{\lambda_{17}}{2}\upsilon_\eta^2 - \frac{\lambda_{20}}{2}\left(\frac{\upsilon_\eta\upsilon_\rho\upsilon_\Delta}{\upsilon_\Delta}\right) - M_1\frac{\upsilon_\eta^2}{\upsilon_\Delta} = 0$$

$$\mu_S^2 + \frac{\lambda_{10}}{2}\upsilon_\Delta^2 + \frac{\lambda_{11}}{2}(\upsilon_\sigma^2 + \upsilon_\Delta^2) + \frac{\lambda_{12}}{2}\upsilon_\eta^2$$

$$+ \frac{\lambda_{13}}{2}\upsilon_\rho^2 + \frac{\lambda_{14}}{2}\upsilon_{x'}^2 + \lambda_{15}\frac{\upsilon_\eta^2\upsilon_\sigma}{\upsilon_\Delta} + \quad (34)$$

$$\frac{\lambda_{16}}{2}\upsilon_\eta^2 - M_2\frac{\upsilon_{x'}^2}{\upsilon_\sigma} + \frac{\lambda_{21}}{2}\left(\frac{\upsilon_\eta\upsilon_\rho\upsilon_\Delta}{\upsilon_\Delta}\right) = 0$$

In any conventional seesaw mechanism, the masses of the particles inherent of the mech-anism and the energy scale associated to the violation of the lepton number both must lie in the GUT regime. Bringing this

to our mechanism, we have that the masses of the scalars that compose the sextet, μ_S and the energy scale M_1 and M_2 that appear in terms that violated explicitly the lepton number both must lie in the GUT energy regime which is around 10^{12}GeV. For sake of simplicity, here we assume $\mu_S \approx M_1 \approx M_2 = M$.

As $\upsilon_\sigma, \Delta << \upsilon_{x',\rho,\eta}$ and $\upsilon_{x',\rho,\eta} << M$, we see that the fourth expression in Equation 34 provides $\upsilon_\Delta = \frac{\upsilon_\eta^2}{M}$ while the last one provides $\upsilon_\sigma = \frac{\upsilon_{x'}^2}{M}$. On substituting these expressions for the VEV's υ_Δ and υ_σ in Equation 32, we obtain:

$$m_{\nu L} = G\frac{\upsilon_\eta^2}{M}, m_{\nu R} = G\frac{\upsilon_{x'}^2}{M} \quad (35)$$

Notice that, the higher the M, the smaller the masses of ν_L and ν_R. This is our type II seesaw mechanism where the masses of the neutrinos are suppressed by the high energy scale M. To go further and make some predictions, there are no other way unless take fine-tunning of the free parameters involved in the mechanism. However, note that, apart from the Yukawa coupling G_{ab}, we have that $\frac{m_{\nu_b}}{m_{\nu_L}} = \frac{\upsilon_{x'}^2}{\upsilon_\eta^2}$. For typical values of both VEV's, for example, $\upsilon_\eta = 10^2$GeV and $\upsilon_{x'} = 10^4$GeV we obtain $\frac{m_{\nu_b}}{m_{\nu_L}} = 10^4$. Thus, for left-handed neutrinos of mass of order of 10^{-1}eV we may have sterile neutrinos of mass of few KeV. This is an encouraging result because sterile neutrinos with mass in this range is a viable warm dark matter candidate (Dodelson and Widrow, 1994).

Simple Realization of the Inverse Seesaw Mechanism

The Inverse Seesaw (ISS) mechanism is a phenomenological small mass generation mecha-nism for the neutrinos since it works at TeV scale and then may be probed with the present colliders as LHC (Mohapatra, 1986).

For the implementation of the ISS mechanism in the 3-3-1 model with right-handed neutrinos we need to enlarge the leptonic sector of the model by adding three singlet neutral fermions (Dias et al., 2005):

$$f_{aL} = (\nu_{aL}\ e_{aL}\ V_{aL}^C)^T \sim (3,-1/3),$$
$$e_{aR} \sim (1,-1)\ N_{aL} \sim (1,0) \quad (36)$$

where, a = 1, 2, 3 (Catao et al., 2012).

The relevant Yukawa Lagrangian for the lepton sector that yields the ISS mechanism for the neutrinos is composed by the following summation of terms:

$$L_{ISS}^Y = G_{ab} \, \epsilon_{ijk} \, \overline{L^C_{ai}} \rho^*_j L_{bk} + G'_{ab} \overline{L}_a X (N_{bL})^C + \frac{1}{2} \overline{N}^C_L \mu N_L + H.c \quad (37)$$

With the set of VEV's of Equation 22 and considering the basis $S_L = (V_L, V_L^C, N_L)$, the mass terms above can be cast in the following manner:

$$L_{mass} = \frac{1}{2} \overline{S}^C_L M_\nu S_L + H.c \quad (38)$$

With the mass matrix M_ν having the texture:

$$M_\nu = \begin{pmatrix} 0 & m_M^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix} \quad (39)$$

where, the 3×3 matrices are defined as:

$$M_{ab} = G'_{ab} \frac{\upsilon_{x'}}{\sqrt{2}} \quad (40)$$

$$MD_{ab} = G'_{ab} \frac{\upsilon_{x'}}{\sqrt{2}} \quad (41)$$

With M_{ab} and m_{Dab} being Dirac mass matrices, with this last one being anti-symmetric. The mass matrix in Equation (39) is characteristic of the ISS mechanism. We would like to call the attention to the fact that the two energy scales related with the model's gauge symmetry breakdown appear in the mass matrix. Namely, $\upsilon_{x'}$ in M_{ab} is connected with $SU(3)_L \otimes U(1)_N / SU(2)_L \otimes U(1)_Y$ and could be expected to be at the TeV scale leading to observable effects at the LHC, while υ_ρ in m_{Dab} is connected with the electroweak standard model symmetry breakdown scale. The third scale of energy, μ, is a energy scale characteristic of the ISS mechanism and lie at KeV scale.

The mass matrix above is the typical one of the ISS mechanism whose diagonalization leads to three light Majorana neutrinos given by:

$$m_{light} = m_D^T M^{-1} \mu (M^T)^{-1} m_D \quad (42)$$

And six heavy new neutrinos with masses around TeV scale.

Returning to m_{light}, on substituting $m_D = G\upsilon_\rho$, $M = G'\upsilon_{x'}$, we obtain:

$$m_{light} = (G^T (G'^T)^{-1} \mu (G')^{-1} G) \frac{\upsilon_\rho^2}{\upsilon_{x'}^2} \quad (43)$$

Remember that G is an anti-symmetric matrix, implying that one eigenvalue of the neutrino mass matrix in equation 43 IS NULL. FOR $\upsilon_\rho \simeq 10^2 \, \text{GeV}, \upsilon_{x'} \simeq 10^3 \, \text{GeV}$ and $\mu \simeq 10^{-7} \, \text{GeV}$, we obtain two light neutrinos with masses around eV.

Conclusion

The explanation of the lightness of the neutrino masses is a challenge for any gauge theory. This is also the case for models based on the 3-3-1 gauge symmetry. This is particularly true for the case of the minimal 3-3-1 model where perturbative regime is trustable until 4-5 TeV. We know that the most easy and elegant way of generating small neutrino masses is supposing they are consequence of physics at GUT scale. This idea is realized through seesaw mechanisms. The difficulty of the minimal 3-3-1 model to generate small neutrino masses is that there effective dimension-5 operators exist and are suppressed by an energy scale of order of 5 TeV (Dong and Si, 2014). Thus, as a first guess we looked for discrete symmetries that avoid dangerous effective operators of low order. This was done in (Queiroz *et al*., 2010) by imposing a Z_3 symmetry with a special representation. Other possibility, which do not need to worry with effective dimension-5 operators, was presented in (Caetano *et al*., 2012) where a combination of type I with type II seesaw mechanisms where used to obtain theoretically consistent model of small neutrino masses.

In regard to the 3-3-1 with right-handed neutrinos, differently from the minimal 3-3-1 case, effective dimension-five operators are suffcient to generate light neutrino masses. However, as the model has the right-handed neutrinos composing, together with the left-handed neutrinos, the same leptonic triplet, then, as nice result, the model provides left-handed and right-handed light neutrinos. The effective dimension-5 operator may be realized through a kind of type II seesaw mechanism implemented by a sextet of scalars belonging to the GUT scale. The interesting point here is that the left-handed as well as the right-handed neutrino both develop light masses with the right-handed ones having masses at KeV scale In view of this, the lightest of them may be stable and constitute some portion of the dark matter of the universe in the form warm dark matter. For completeness reasons, we would like to call the attention to the fact that, as far as I know, minimal versions of the 3-3-1 models do not contain in their particle spectrum any candidate that fulfill the condition required to be a dark matter candidate, while 3-3-1 models with right-handed neutrinos present dark matter candidate in the fermion and scalar forms (Pires and Silva, 2007).

Another positive aspect concerning the 3-3-1 model with right-handed neutrinos is that it realizes naturally the inverse seesaw mechanism. Thus, we have a model with a seesaw mechanism that may be probed at the LHC and future colliders once such a mechanism works at TeV scale. Thus, in what concern neutrino masses, the 3-3-1 model with right- handed neutrinos seems to be more attractive than the minimal 3-3-1 model.

In what concern LHC and 3-3-1 signature, many things remain to be done. Apart from some works considering the Higgs phenomenology (Alves *et al.*, 2011), we have constraint on the Z0 mass being abobe 2TeV (Coutinho *et al.*, 2013) and some phenomenological aspects considering charged scalar production (Cogollo *et al.*, 2014).

Acknowledgment

Ethics

I address any ethical issues that may arise after the publication of this mechanism.

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