

Original Research Paper

Derivation of Potentially Important Masses for Physics and Astrophysics by Dimensional Analysis

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Abstract: The Hubble constant has been added in addition to the three fundamental constants (speed of light, gravitational constant and Planck constant) used by Max Planck, for derivation of the Planck mass by dimensional analysis. As a result, a general solution is found for the mass dimension expression $m = \gamma^p m_0$, where $m_0 \equiv m_p$ is the Planck mass, $\gamma = 1.23 \times 10^{-61}$ is a small dimensionless quantity and p is an arbitrary parameter in the interval $[-1, 1]$. The Planck mass $m_p = 2.17 \times 10^{-8}$ kg, mass of the Hubble sphere $M_H \sim 10^{53}$ kg, minimum quantum of mass/energy $m_G = 2.68 \times 10^{-69}$ kg, Weinberg mass $m_W = 1.08 \times 10^{-28}$ kg, mass of hypothetical quantum gravity atom $M_G = 3.8 \times 10^{12}$ kg, Eddington mass limit of stars $M_E = 6.6 \times 10^{32}$ kg and some more masses potentially important for the physics and astrophysics represent particular solutions for values of p , expressed as fractions with small numerators and nominators.

Keywords: Dimensional Analysis, Fundamental Constants, Ockham's Razor, Mass of the Hubble Sphere, Weinberg Mass, Eddington Mass Limit

Introduction

The dimensional analysis is a conceptual tool often applied in physics and astrophysics to understand physical situations involving certain physical quantities (Bridgman, 1922; Kurth, 1972; Bhaskar and Nigam, 1990; Petty, 2001). It is routinely used to check the plausibility of the derived equations and computations. When it is known with which other determinative quantities a particular quantity would be connected, but the form of this connection is unknown, a dimensional

equation $[q_0] \sim \prod_{i=1}^n [q_i]^{n_i}$ is composed for its finding. In

the left side of the equation is placed the unit of this quantity q_0 with its dimensional exponent and in the right side of the equation is placed the product of units of the determinative quantities q_i raised to the unknown exponents n_i , where n is a positive integer and the exponents n_i are rational numbers. Most often, the dimensional analysis is applied in the mechanics, aerodynamics, astrophysics and other fields of the modern physics, where there are many problems with a few determinative quantities.

The Planck mass as defined by Planck (1906) in terms of three fundamental constants, speed of light in

vacuum (c), gravitational constant (G) and reduced Planck constant \hbar , is $m_p = \sqrt{\frac{\hbar c}{G}}$. Since the constants c ,

G and \hbar represent three very basic aspects of the universe (i.e. the relativistic, gravitational and quantum phenomena), the Planck mass appears to a certain degree a unification of these phenomena. The Planck mass has many important theoretical ramifications in modern physics. One of them is that the energy equivalent of

Planck mass $E_p = m_p c^2 = \sqrt{\frac{\hbar c^5}{G}} \sim 10^{19}$ GeV appears to be

the unification energy of the fundamental interactions (Georgi *et al.*, 1974). Additionally, the Planck mass can be derived approximately by defining it as a mass whose Compton wavelength and Schwartzchild radius are equal (Bergmann, 1992).

The Hubble constant H has been added to the set of constants c , G , \hbar and thus a unique mass dimension quantity has been derived for every triad (c , G , H), (c , \hbar , H) and (G , \hbar , H) by dimensional analysis (Valev, 2013). Thus, three new fundamental masses are found, i.e.

$$m_1 = \frac{c^3}{GH} \sim 10^{53} \text{ kg}, \quad m_2 = \frac{\hbar H}{c^2} = 2.68 \times 10^{-69} \text{ kg} \quad \text{and}$$

$$m_3 = \sqrt[5]{\frac{H \hbar^3}{G^2}} = 1.43 \times 10^{-20} \text{ kg}. \quad \text{The mass } m_1 \text{ is identified}$$

with the mass of the Hubble sphere $M_H = \frac{c^3}{2GH} = 8.8 \times 10^{52} \text{ kg}$, m_2 with minimum quantum of mass/energy and m_3 is conjectured to be the mass of a yet unknown super heavy particle or fundamental energetic scale. According to the contemporary cosmology, the Hubble constant slowly decreases with the age of the universe $\dot{H}/H \sim -H = -2.3 \times 10^{-18} \text{ s}^{-1}$, i.e. H decreased only 33% from the Solar system formation till now.

In the present work we seek a mass dimension quantity that represents as a product of rational exponents of the four constants – c , G , \hbar and H .

General Solution of the Problem for Finding of a Mass Dimension Quantity by Means of Fundamental Constants c , G and H

By dimensional analysis, we search for a mass dimension quantity m in the form of product of rational exponents n_1, n_2, n_3 and n_4 of the constants c, G, \hbar and H :

$$m = kc^{n_1} G^{n_2} \hbar^{n_3} H^{n_4} \sim c^{n_1} G^{n_2} \hbar^{n_3} H^{n_4} \quad (1)$$

The exponents n_1, n_2, n_3 and n_4 are unknown quantities that can be found by matching dimensions on both sides of equation (1) and k is a dimensionless parameter (coefficient) on the order of unity.

Replacing dimensions of m, c, G, \hbar and H in (1) we find the dimensional equation:

$$L^0 T^0 M^1 = L^{n_1+3n_2+2n_3} T^{-n_1-2n_2-n_3-n_4} M^{-n_2+n_3} \quad (2)$$

From Equation (2) we find system of linear equations for unknown quantities n_1, n_2, n_3 and n_4 :

$$\begin{aligned} n_1 + 3n_2 + 2n_3 &= 0 \\ -n_1 - 2n_2 - n_3 - n_4 &= 0 \\ -n_2 + n_3 &= 1 \end{aligned} \quad (3)$$

The rank of augmented matrix of the system $r = 3$ is equal to the rank of the coefficient matrix. Thus, in accordance with the Rouche-Capelli theorem the system is consistent and so must have at least one solution. The solution is unique if and only if the rank equals the number of variables. In the system (3) the number of variables $m = 4 > r = 3$, therefore the solution is not unique, but having infinitely many solutions. However, upon introducing the concept of a free parameter p , where in it is accepted that $n_4 = p$, system (3) can be transform to:

$$\begin{aligned} n_1 + 3n_2 + 2n_3 &= 0 \\ -n_1 - 2n_2 - n_3 &= p \\ -n_2 + n_3 &= 1 \end{aligned} \quad (4)$$

The determinant of system (4) is $\Delta = 2 \neq 0$ and the system has a solution that is dependent upon a free

parameter p . We find the solution of the system (4) by means of Cramer's rule:

$$\begin{aligned} n_1 &= (1-5p)/2, \\ n_2 &= (p-1)/2, \\ n_3 &= (p+1)/2, \\ n_4 &= p \end{aligned} \quad (5)$$

where, p is a free parameter.

Replacing the solution (5) in Equation (1) we find Equation (6) for the mass m :

$$m \sim c^{(1-5p)/2} G^{(p-1)/2} \hbar^{(p+1)/2} H^p \quad (6)$$

Obviously, the Equation (6) can be transformed in Equation (7):

$$m \sim c^{\frac{1-5p}{2}} G^{\frac{p-1}{2}} \hbar^{\frac{p+1}{2}} H^p = \sqrt{\frac{c\hbar}{G}} \left(\sqrt{\frac{G\hbar H^2}{c^5}} \right)^p \quad (7)$$

Therefore, we find the general solution (8):

$$m \sim \gamma^p m_0 \quad (8)$$

where, $\gamma = \sqrt{\frac{G\hbar H^2}{c^5}} \sim 10^{-61}$ is exceptionally small

dimensionless quantity, $m_0 \equiv m_p = \sqrt{\frac{\hbar c}{G}} = 2.17 \times 10^{-8} \text{ kg}$ is the Planck mass and p is a free parameter.

Although the parameter p can take arbitrary values in the interval $(-\infty, +\infty)$, only solutions in the interval $[-1, 1]$ could have physical meaning, because for limit values $p = -1$ and $p = 1$ the resulting solutions are, respectively, the mass of the Hubble sphere $M_H \sim m_1 = \frac{c^3}{GH} = 1.76 \times 10^{53} \text{ kg}$ (Carvalho, 1995; Valev, 2009) and the minimum measurable mass/energy in the universe $m_2 \equiv m_G = \frac{\hbar H}{c^2} = 2.68 \times 10^{-69} \text{ kg} = 1.5 \times 10^{-33} \text{ eV}$ (Sivaram, 1982; Alfonso-Faus, 2012). The exceptionally small mass m_3 seems close to the graviton mass obtained by different methods (Woodward *et al.*, 1975; Gershtein *et al.*, 1998; Valev, 2008; Alves *et al.*, 2011).

According to Ockham's razor principle, all other things being equal, the simplest theory is the most likely to be true (Rodriguez-Fernandez, 1999). In science, this principle is used as a heuristic technique (discovery tool) to guide scientists in the development of theoretical models (Gauch, 2003). Therefore, in the following section, we consider particular solutions where the free parameter $|p| \leq 1$ represents as a fraction with a small numerator and denominator,

i.e. $p = 0, \pm 1, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{5}, \pm \frac{2}{3}$. We will show that some such solutions result in mass formulas that could be interesting for contemporary particle physics and astrophysics.

Particular Solutions where the Free Parameter $|p| \leq 1$ Represents as Fractions having Small Numerators and Denominators

From the general solution (8) we find the Planck mass m_p as a particular solution (9) in case of $p = 0$:

$$m_0 \equiv m_p = \sqrt{\frac{\hbar c}{G}} = 2.17 \times 10^{-8} \text{ kg} \quad (9)$$

As it has been mentioned in Section 2, from Equation (8) we find mass of the Hubble sphere M_H as a particular solution (10) at $p = -1$:

$$M_H \sim m_1 = \frac{c^3}{GH} = 1.76 \times 10^{53} \text{ kg} \quad (10)$$

Analogously, from the general solution (8) we find the minimum quantum of mass/energy m_G as a particular solution (11) at $p = 1$:

$$m_2 \equiv m_G = \frac{\hbar H}{c^2} = 2.68 \times 10^{-69} \text{ kg} = 1.5 \times 10^{-33} \text{ eV} \quad (11)$$

From Equation (8) we find the particular solution (12) at $p = \frac{1}{3}$:

$$m_4 \equiv m_w = c^{-\frac{1}{3}} G^{-\frac{1}{3}} \hbar^{\frac{2}{3}} H^{\frac{1}{3}} = \sqrt[3]{\frac{\hbar H^2}{cG}} = 1.08 \times 10^{-28} \text{ kg} = 60.8 \text{ MeV} \quad (12)$$

The Equation (12) represents the well known Weinberg mass formula (Weinberg, 1972). The physical meaning of the Weinberg mass was found from Sivaram (1982). He shows the Weinberg mass represents the lightest mass whose self-gravitational energy has measurable value for the time of existence of the universe $H^{-1} \approx 1.38 \times 10^{10}$ years.

From the general solution (8) we find the particular solution (13) at $p = -\frac{1}{3}$:

$$m_5 = c^{\frac{4}{3}} G^{-\frac{2}{3}} \hbar^{\frac{1}{3}} H^{-\frac{1}{3}} = c \cdot \sqrt[3]{\frac{c\hbar}{G^2 H}} = 4.36 \times 10^{12} \text{ kg} \quad (13)$$

It has been shown (Forsythe, 2009) that the hypothetical ‘Quantum gravity atom’ built up from a

central electro-neutral mass M_G around which orbits an electro-neutral particle of electron mass m_e at a distance equal to the Bohr radius $a_0 = \frac{\hbar^2}{m_e e^2} = 5.3 \times 10^{-11} \text{ m}$, possesses gravitational potential $V = \frac{GM_G m_e}{a_0}$ equal to the electrostatic potential $V_E = \frac{e^2}{a_0}$. From here Forsythe finds the central mass M_G :

$$M_G = \frac{e^2}{Gm_e} = 3.8 \times 10^{12} \text{ kg} \sim m_5 \quad (14)$$

From the general solution (8) we find the particular solution (15) at $p = \frac{1}{2}$:

$$m_6 = c^{-\frac{3}{4}} G^{-\frac{1}{4}} \hbar^{\frac{3}{4}} H^{\frac{1}{2}} = \sqrt[4]{\frac{\hbar^3 H^2}{Gc^3}} = 7.64 \times 10^{-39} \text{ kg} = 4.3 \times 10^{-3} \text{ eV} \quad (15)$$

Obviously, the mass m_6 , obtained from Equation (8) at $p = \frac{1}{2}$ is of the order of the neutrino rest mass (Goobar *et al.*, 2006).

From (8) we find the particular solution (16) at $p = -\frac{1}{2}$:

$$m_7 = c^{\frac{7}{4}} G^{-\frac{3}{4}} \hbar^{\frac{1}{4}} H^{-\frac{1}{2}} = c \cdot \sqrt[4]{\frac{c^3 \hbar}{G^3 H^2}} = 6.18 \times 10^{22} \text{ kg} \quad (16)$$

The mass m_1 represents approximately 1% of the Earth mass $M_{Earth} = 5.97 \times 10^{24} \text{ kg}$ and is close to the Moon mass $M_{Moon} = 7.34 \times 10^{22} \text{ kg}$, that presents typical satellite in the Solar system.

From the general solution (8) we find the particular solution (17) at $p = \frac{1}{4}$:

$$m_8 = c^{-\frac{1}{8}} G^{-\frac{3}{8}} \hbar^{\frac{5}{8}} H^{\frac{1}{4}} = \sqrt[8]{\frac{\hbar^5 H^2}{cG^3}} = 1.29 \times 10^{-23} \text{ kg} = 7.25 \text{ TeV} \quad (17)$$

This energy is typical for the energy of protons in Large Hadron Collider (LHC) and possibly is connected with mass of yet unobserved heavy particle or fundamental energetic scale.

From (8) we find the particular solution (18) at $p = -\frac{1}{4}$:

$$m_9 = c^{\frac{9}{8}} G^{-\frac{5}{8}} \hbar^{\frac{3}{8}} H^{-\frac{1}{4}} = c \cdot \sqrt[8]{\frac{c\hbar^3}{G^5 H^2}} = 3.67 \times 10^7 \text{ kg} \quad (18)$$

The mass m_9 most probably has no reference to the fundamental physics.

From the general solution (8) we find the particular solution (19) at $p = -\frac{1}{5}$:

$$m_{10} = cG^{-\frac{3}{5}}\hbar^{\frac{2}{5}}H^{-\frac{1}{5}} = c \cdot \sqrt[5]{\frac{\hbar^2}{G^3H}} = 3.4 \times 10^4 \text{ kg} \quad (19)$$

The mass m_{10} hardly have some physical meaning.

The case $p = \frac{1}{5}$ uniquely yields the mass equation (20):

$$m_3 = \sqrt[5]{\frac{H\hbar^3}{G^2}} = 1.43 \times 10^{-20} \text{ kg} = 8.0 \times 10^6 \text{ GeV} \quad (20)$$

This mass also can't be identified but possibly could be considered a heuristic prediction of the suggested model for a super heavy unobserved particle or

fundamental energetic scale intermediate for electroweak scale $\sim 250 \text{ GeV}$ and GUT scale $\sim 10^{16} \text{ GeV}$.

From (8) we find the particular solution (21) at $p = \frac{2}{3}$:

$$m_{11} = c^{-\frac{7}{6}}G^{-\frac{1}{6}}\hbar^{\frac{5}{6}}H^{\frac{2}{3}} = \frac{1}{c} \cdot \sqrt[6]{\frac{\hbar^5 H^4}{cG}} \quad (21)$$

$$= 5.39 \times 10^{-49} \text{ kg} = 3.0 \times 10^{-13} \text{ eV}$$

This mass is close to one of the seven fundamental equidistant masses found in (Forsythe and Valev, 2014), namely the mass $M_{(-1)} = 7.15 \times 10^{-49} \text{ kg}$.

Finally, from the general solution (8) we find the particular solution (22) at $p = -\frac{2}{3}$:

$$m_{12} = c^{\frac{13}{6}}G^{-\frac{5}{6}}\hbar^{\frac{1}{6}}H^{-\frac{2}{3}} = c^2 \cdot \sqrt[6]{\frac{c\hbar}{G^5 H^4}} \quad (22)$$

$$= 8.76 \times 10^{32} \text{ kg}$$

Table 1. Masses whose free parameters are in the range $|p| \leq 1$ and appear in the general solution as fractions having small numerators and denominators

p	Mass corresponding to p	Identification of mass m_i
-1	$m_1 = \frac{c^3}{GH} = 1.76 \times 10^{53} \text{ kg}$	Mass of the Hubble sphere M_H
$-\frac{2}{3}$	$m_{12} = c^2 \cdot \sqrt[6]{\frac{c\hbar}{G^5 H^4}} = 8.76 \times 10^{32} \text{ kg}$	Eddington mass limit of stars M_E
$-\frac{1}{2}$	$m_7 = c \cdot \sqrt[4]{\frac{c^3 \hbar}{G^3 H^2}} = 6.18 \times 10^{22} \text{ kg}$	Mass of the Moon (Typical satellite in the Solar system)
$-\frac{1}{3}$	$m_5 = c \cdot \sqrt[3]{\frac{c\hbar}{G^2 H}} = 4.36 \times 10^{12} \text{ kg}$	Mass of 'Quantum Gravity Atom' M_G
$-\frac{1}{4}$	$m_9 = c \cdot \sqrt[8]{\frac{c\hbar^3}{G^5 H^2}} = 3.67 \times 10^7 \text{ kg}$	-
$-\frac{1}{5}$	$m_{10} = c \cdot \sqrt[5]{\frac{\hbar^2}{G^3 H}} = 3.40 \times 10^4 \text{ kg}$	-
0	$m_0 = \sqrt{\frac{c\hbar}{G}} = 2.17 \times 10^{-8} \text{ kg}$	Planck mass m_p
$\frac{1}{5}$	$m_3 = \sqrt[5]{\frac{H\hbar^3}{G^2}} = 1.43 \cdot 10^{-20} \text{ kg}$	Prediction for super massive unknown particle
$\frac{1}{4}$	$m_8 = \sqrt[8]{\frac{\hbar^5 H^2}{cG^3}} = 1.29 \times 10^{-23} \text{ kg}$	Prediction for massive unknown particle
$\frac{1}{3}$	$m_4 = \sqrt[3]{\frac{H\hbar^2}{cG}} = 1.08 \times 10^{-28} \text{ kg}$	Weinberg mass m_W
$\frac{1}{2}$	$m_6 = \sqrt[4]{\frac{\hbar^3 H^2}{Gc^3}} = 7.64 \times 10^{-39} \text{ kg}$	Neutrino mass m_ν
$\frac{2}{3}$	$m_{11} = \frac{1}{c} \cdot \sqrt[6]{\frac{\hbar^5 H^4}{cG}} = 5.39 \times 10^{-49} \text{ kg}$	Forsythe-Valev mass $M_{(-1)}$
1	$m_2 = \frac{\hbar H}{c^2} = 2.68 \times 10^{-69} \text{ kg}$	Minimum quantum of mass/energy (Graviton mass m_G)

The mass m_{12} has been identified in (Forsythe and Valev, 2014) with Eddington mass limit of the most massive stars $M_E = 6.6 \times 10^{32}$ kg.

The above derived masses, whose free parameters are in the range $|p| \leq 1$ and appear in the general solution as fractions having small numerators and denominators, are presented in Table 1.

Probably, the general solution (8) includes additional masses interesting from the physical view point, but indefiniteness of the parameter p doesn't allow unambiguous finding of these masses.

Time dependence of some derived masses is natural and clear. For example mass of the Hubble sphere $M_H = \rho_c V_H = \frac{3H^2}{8\pi G} \frac{4\pi}{3} \frac{c^3}{H^3} = \frac{c^3}{2GH} \approx m_1$ increase with cosmological expansion because of faster increase of volume (cubic) than decrease of total density $\bar{\rho} = \rho_c$ (quadratic). Same is valid for the minimum quantum of mass/energy $m_2 \equiv m_G = \frac{\hbar H}{c^2} \sim 10^{-33}$ eV and Weinberg mass

$$m_4 \equiv m_w = \sqrt[3]{\frac{H\hbar^2}{cG}} = 60.8 \text{ MeV}$$

because they are related with Heisenberg's uncertainty principle $\Delta E \Delta t \geq \hbar$, where the duration of measurement is limited from the age of the universe $\Delta t \leq H^{-1} = 1.38 \times 10^{10}$ years (Sivaram, 1982). The Planck mass $m_0 \equiv m_p = \sqrt{\frac{\hbar c}{G}} = 2.17 \times 10^{-8}$ kg

include constants c , G and \hbar and is time independent, but the rest derived masses depend from the expansion. In result, the microscopic masses m_2, m_3, m_4, m_6, m_8 and m_{11} decrease with cosmological expansion, while the macroscopic masses $m_1, m_5, m_7, m_9, m_{10}$ and m_{12} and increase with the expansion.

Conclusion

The Hubble constant H has been added to the three fundamental constants (the speed of light in vacuum, Newtonian gravitational constant and reduced Planck constant) used from Max Planck for derivation of Planck mass by dimensional analysis.

We search by dimensional analysis a mass dimension quantity that represents a product of rational exponents of the four constants – c , G , \hbar and H . In result, a general solution has been found of mass dimension quantity

$$m = \gamma^p m_0, \text{ where } m_0 \equiv m_p = \sqrt{\frac{\hbar c}{G}} = 2.17 \times 10^{-8} \text{ kg}$$

is the Planck mass, $\gamma = \sqrt{\frac{G\hbar H^2}{c^5}} = 1.23 \times 10^{-61}$ is a small dimensionless quantity and p is an arbitrary parameter in the interval $[-1, 1]$. According to Ockham's razor principle, all other things being equal, the simplest theory is the most likely to be true. Therefore, we

consider particular solutions where the free parameter $|p| \leq 1$ represents as a fraction with a small numerator and denominator, i.e. $p = 0, \pm 1, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{5}, \pm \frac{2}{3}$.

In result, it has been found that the Planck mass

$$m_p = \sqrt{\frac{\hbar c}{G}} = 2.17 \times 10^{-8} \text{ kg, mass of the Hubble sphere}$$

$$m_H = \frac{c^3}{GH} \sim 10^{53} \text{ kg, minimum quantum of mass/energy}$$

$$m_G = \frac{\hbar H}{c^2} = 2.68 \times 10^{-69} \text{ kg, Weinberg mass}$$

$$m_w = \sqrt[3]{\frac{H\hbar^2}{cG}} = 1.08 \times 10^{-28} \text{ kg, mass of hypothetical}$$

quantum gravity atom $M_G = 3.8 \times 10^{12}$ kg, Eddington mass limit of stars $M_E = 6.6 \times 10^{32}$ kg and some more masses potentially important for the physics and astrophysics represent particular solutions for values of p , expressed as fractions with small numerators and denominators. Likely, some of unidentified masses could have heuristic meaning for astrophysics and high energy physics.

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Ethics

This article is original and contains unpublished material. The corresponding author confirms that all of the other authors have read and approved the manuscript and no ethical issues involved.

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